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Height-Diameter Relationships for *Pinus brutia* Ten. in the Adana-Karaisali Region of Türkiye

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Abstract: In this study, it was aimed to develop height (*H*)-diameter (*D*) models for pure *Pinus brutia* Ten. stands distributed in the Adana-Karaisalı region in Türkiye. Diameter at breast height and tree height data were obtained from 1081 sample trees in 82 sample plots. Additionally, measurements were made regarding various stand variables. Based on the data obtained, parameters were estimated for four base *H-D* models, which provide estimates based only on diameter at breast height, and five generalized *H-D* models, which incorporate stand variables in addition to diameter at breast height.
According to the results of nonlinear regression analysis, models with all parameters being statistically significant were evaluated based on six statistical criteria. The ranking results showed that the Mirkovich (1958)'s generalized *H-D* model achieved the best predictive results, with an coefficient of determination (R^2) value of 0.941, indicating that the model accounts for 94.1% of the variation in tree height and can be considered a reliable tool for estimating tree height in the studied stands. As a result of statistical and graphical evaluations, it can be concluded that generalized *H-D* models perform better and yield more unbiased predictions.

Keywords: Brutian pine, stand variables, diameter at breast height, tree height, nonlinear models

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Introduction

In Türkiye, Brutian pine (*Pinus brutia* Ten.) ranks first among coniferous species in terms of distribution, achieving the widest range in the world within this country. Brutian pine is recognized as a symbol of the Mediterranean region due to its notable drought tolerance. In Türkiye, Brutian pine is primarily distributed in the Eastern Mediterranean basin and also occurs in pure and mixed stands in the Aegean, Southern Marmara, and Western Black Sea regions (Selik, 1963). Brutian pine occupies approximately 5.2 million hectares in Türkiye, of which 3.4 million hectares are productive and 1.8 million hectares are classified as degraded (OGM, 2020). For this reason, it is a species of considerable importance both economically and ecologically.

Tree diameter and height are fundamental variables in forest inventory studies. These variables are utilized as fundamental variables in various forestry applications, including the estimation of the growing stock, developing growth models, assessing stand structural diversity, and creating biomass and carbon stock estimation models. Measuring tree diameters at breast height can be done rapidly, efficiently, and accurately. In contrast, inventorying tree height is a more time-consuming and challenging process, requiring greater effort and resources (Arabatzis &

Burkhart, 1992; Huang et al., 2000; Sharma & Parton, 2007; Diamantopoulou & Özçelik, 2012; Ercanlı, 2015; Sağlam & Sakici, 2024). Particularly in practice, single- and double-entry volume equations are used for volume estimation of stands and individual trees, necessitating independent variables such as diameter and height for their application. However, while single-entry volume equations are practical, it is more appropriate to favor double-entry volume equations for reliable and accurate volume estimates. This is because single-entry volume equations assume that trees with the same diameter but different height values have the same volume (Sakici et al., 2018; Zaifoğlu & Sağlam, 2024; Boz & Sağlam, 2025). Therefore, height (*H*)-diameter (*D*) models are crucial for reliable predictions in both scientific research and forest management practices.

Linear and nonlinear regression models are used to model the relationship between diameter and height of trees. Linear models are used in applications that do not require high-precision estimates. In contrast, nonlinear models, which are more flexible and offer greater accuracy, are more frequently preferred (Larsen & Hann, 1987; Arabatzis & Burkhart, 1992). This preference for accurate modeling is especially important since *H-D* models are a key component of the ecosystem-based functional planning approach used in forest management plans in Türkiye (Özçelik & Çapar, 2014).

The relationship between a tree's diameter and height varies from stand to stand, and this relationship can change over time within the same stand, as the *H-D* relationships of stands differ with stand age, density, and site productivity (Curtis, 1967; Castedo-Dorado et al., 2006). Additionally, growth patterns are significantly influenced by climatic factors (Filipiak & Napierała-Filipiak, 2008). Due to the heterogeneous nature of stand structures in forest ecosystems, it is quite challenging to explain *H-D* relationships using a single model. Therefore, to mitigate the variability in *H-D* relationships, separate *H-D* models should be developed for each stand (Calama & Montero, 2004; Özçelik & Çapar, 2014). At the same time, to minimize this level of variance, stand variables can be included as independent variables in *H-D* models (Curtis, 1967; Larsen & Hann, 1987; Temesgen & Gadow, 2004; Castedo-Dorado et al., 2006). In addition to the diameter at breast height of trees, equations that include various stand characteristics as independent variables are referred to as "generalized height-diameter models" (Mısır, 2010; Özçelik & Çapar, 2014; Ercanlı & Eyüboğlu, 2019).

Research on *H-D* models, which play a crucial role in forest modeling and practices, is of significant importance. Researches related to generalized *H-D* models in Türkiye include the studies of Sönmez (2009), Mısır (2010), Ercanlı et al. (2012), Özçelik et al. (2013, 2014, 2018), Özçelik and Çapar (2014), Ercanlı (2015, 2019, 2020), Çatal and Carus (2018), Ercanlı and Eyüboğlu (2019), Bolat et al. (2022), Seki and Sakici (2022), Şen and Sağlam (2024) and Sağlam and Sakici (2024). Among the studies conducted, the research by Özçelik and Çapar (2014) and Çatal and Carus (2018) focused on the Brutian pine species and was carried out in the Western Mediterranean region. Research on *H-D* models in Brutian pine stands, which is the most widely distributed conifer species in Türkiye, is particularly important in the Eastern Mediterranean region, one of the key areas for this species. In this context, this study holds significant importance as it will be the first to comprehensively investigate the *H-D* relationships in Brutian pine stands in the Eastern Mediterranean region.

This study aims to develop *H-D* models for Brutian pine stands in the Adana-Karaisalı region. To this end, four base *H-D* models and five generalized *H-D* models, which are commonly used in the literature and also exhibiting flexible structures, have been selected (Çatal & Carus, 2018; Stankova et al., 2022; Sağlam & Sakici, 2024). The fact that the Brutian pine species has the widest distribution in the world and ranks first among conifer species in terms of distribution highlights the importance of this species. Approximately 60% of the Karaisalı Forest Enterprise area consists of forested land, with the Brutian pine species being the most significant species in this study area. The richness of forested areas in the study region, along with the significant distribution of Brutian pine in these areas, provides an important rationale for the development of *H-D* models.

Materials and methods

Study area

This study focused on the *H-D* relationships of pure Brutian pine stands at the Karaisalı Forest Enterprise (Fig. 1). Within the boundaries of the Karaisalı Forest Enterprise contains 62,544.10 hectares of pure Brutian pine stands, including 55,445.75 hectares of productive forests and 7,098.35 hectares of degraded forests. According to the Thornthwaite and Köppen-Trewartha climate classifications, the study area is characterized by a semi-humid Mediterranean climate type with cool winters and very hot summers (URL-1).

Data

To develop *H-D* models, measurements were conducted on 1081 sample trees taken from a total of

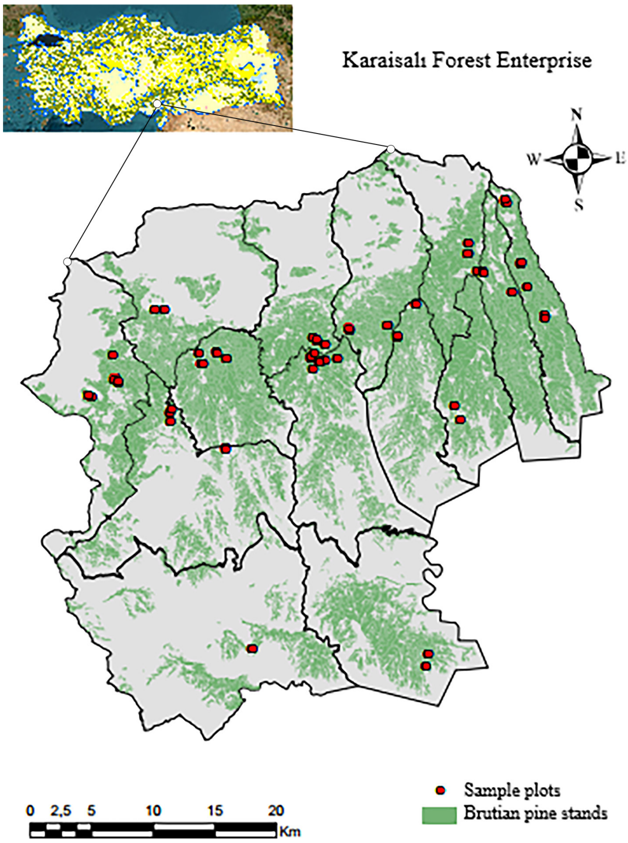

Fig. 1. Sample plots distribution

82 sample plots, distributed across various diameter classes, stand densities, and site qualities. The sample plots are circular and range in size from 400 m² to 1200 m², depending on the stand densities. The measurements conducted in the sample plots are provided follows: (*i*) The diameters at breast height (*D*) of all trees with a diameter of 8 cm or more ($D \geq$ 8 cm) were measured using a caliper with an accuracy of 0.1 cm. (*ii*) The heights (*H*) of approximately 12 to 14 trees of varying diameters and heights were measured with a height measurement device to an accuracy of 0.1 m. (*iii*) In order to determine the dominant height, the heights of the dominant/co-dominant trees in the sample plot were measured according to the calculation of 100 trees per hectare (for example, 6 trees in a 600 m² sample plot), and subsequently, the dominant height (H_0) was calculated. (*iv*) Similarly, the dominant diameter is the average diameter of the trees with the thickest diameter according to the calculation of 100 trees per hectare. (*v*) Quadratic mean diameter is calculated as the square root of the mean of the squares of the diameters of the trees. The distribution of the sample plots within the study area is shown in Fig. 1.

Data Analysis and Candidate Models

The data obtained in this study were randomly divided into two groups: modeling data (880 sample trees; approximately 80% of the sample trees) for the development of the models and testing data (201 sample trees; approximately 20% of the sample trees) for testing the validity of the models. Descriptive statistics about the sample trees and plots in both groups are presented in Table 1, while the *H-D* distributions for the modeling, testing, and total data sets are illustrated in Fig. 2.

Table 1. Descriptive statistics of tree and stand variables for modeling, testing and total data sets

Data		Variables	Minimum	Maximum	Mean	Standard Deviation
Modeling	Tree level	Diameter at breast height (*D*, cm)	8.1	93.0	31.1	13.3
		Tree height (*H*, m)	4.0	30.3	13.2	4.8
	Stand level	Dominant height (H_0, m)	4.80	28.70	14.25	4.84
		Dominant diameter (D_0, cm)	13.1	81.0	36.7	12.9
		Quadratic mean diameter (d_q, cm)	8.8	72.1	30.4	12.2
		Basal area (*G*, m² ha⁻¹)	5.45	101.92	29.96	16.83
		Tree numbers (*N*, trees ha⁻¹)	125	4000.00	540.7	563.9
Testing	Tree level	Diameter at breast height (*D*, cm)	10.0	73.0	31.5	12.1
		Tree height (*H*, m)	4.6	29.0	13.1	4.5
	Stand level	Dominant height (H_0, m)	4.80	27.50	14.32	4.33
		Dominant diameter (D_0, cm)	13.1	78.7	36.8	11.2
		Quadratic mean diameter (d_q, cm)	8.8	70.7	30.4	11.0
		Basal area (*G*, m² ha⁻¹)	5.45	84.89	29.76	15.61
		Tree numbers (*N*, trees ha⁻¹)	125	4000.00	540.8	567.2
Total	Tree level	Diameter at breast height (*D*, cm)	8.1	93.0	31.1	13.1
		Tree height (*H*, m)	4.0	30.3	13.2	4.8
	Stand level	Dominant height (H_0, m)	4.80	28.70	14.26	4.74
		Dominant diameter (D_0, cm)	13.1	81.0	36.7	12.6
		Quadratic mean diameter (d_q, cm)	8.8	72.1	30.4	12.0
		Basal area (*G*, m² ha⁻¹)	5.45	101.92	29.92	16.61
		Tree numbers (*N*, trees ha⁻¹)	125	4000.00	540.7	564.2

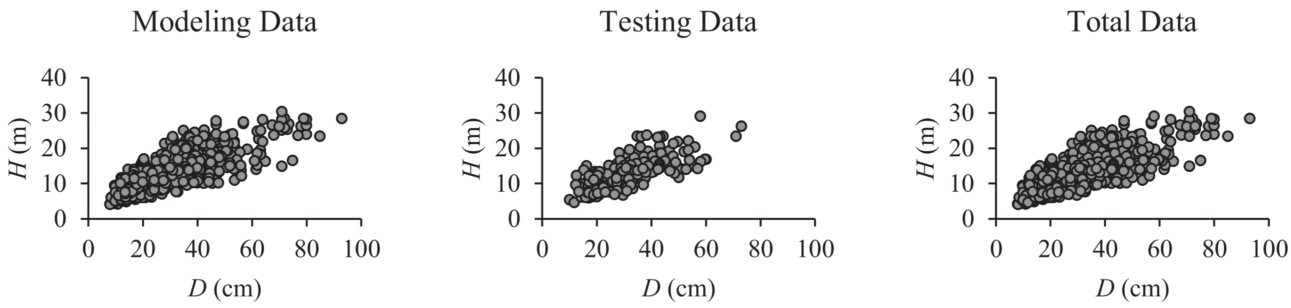

Fig. 2. *H-D* distributions of sample trees

In the scope of this study, four base and five generalized *H-D* models that are frequently referenced in the literature and effectively model *H-D* relationships have been selected. Among these models, the base models provide predictions based on tree diameter, which is relatively easy to measure, while the generalized models incorporate various stand variables alongside tree diameter in their formulations. Model parameters were estimated using nonlinear regression analysis based on the modeling data set. The *H-D* models and their corresponding references are presented in Table 2.

The calculations related to the stand variables included in the models are described below. In order to calculate the dominant heights representing the site quality, the mean of the heights of the dominant or co-dominant trees in the sample plots was taken according to the 100 trees per hectare method in the sample plots. In the sample plots, the mean diameter of the thickest diameter trees was calculated as the dominant diameter (cm) according to the 100 trees per hectare method. Additionally, the quadratic mean diameter (cm), basal area (m^2 ha^{-1}), and tree numbers (trees ha^{-1}) in sample plots were calculated and included as independent variables in the models along with *D*. The 'nls' function in the R statistical software was used to develop the models (R Development Core Team, 2023).

Model evaluation

In the evaluation of the models, six statistical criteria were employed, as outlined below. Among these criteria, high coefficient of determination and low error statistics are desired. Furthermore, model predictions and model residuals were examined graphically.

Coefficient of Determination:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(h_i - \hat{h}_i)^2}{\sum_{i=1}^{n}(h_i - \bar{h}_i)^2}$$

Table 2. H-D model forms

Model	Mathematical Model Forms	References
	Base H-D Models	
M1	$H = (1.3^a + (b^a - 1.3^a)\frac{1 - \exp(-cD)}{1 - \exp(-c\ 100)})^{\frac{1}{a}}$	Schnute (1981)
M2	$H = 1.3 + a(1 - \exp(-bD^c))$	Yang et al. (1978)
M3	$H = 1.3 + \frac{a}{(1 + b^{-1}D^c)}$	Ratkowsky & Reedy (1986)
M4	$H = 1.3 + a(1 - \exp(-bD))^c$	Richards (1959)
	Generalized H-D Models	
M5	$H = (1.3^b + (H_0^b - 1.3^b)\frac{1 - \exp(-aD)}{1 - \exp(-aD_0)})^{\frac{1}{b}}$	Schnute (1981)
M6	$H = 1.3 + (H_0 - 1.3)\frac{\exp(aD^{b+c(H_0-1.3)})}{\exp(aD_0^{b+c(H_0-1.3)})}$	Krumland & Wensel (1988)
M7	$H = aH_0(1 - \exp(\frac{-bD}{d_q}))^c$	Pienaar et al. (1991)
M8	$H = 1.3 + (a + bH_0 - cd_q)(\exp(-\frac{d}{D}))$	Mirkovich (1958)
M9	$H = 1.3 + (aH_0^b)(1 - \exp(-c(\frac{N}{G})^d D))^e$	Richards (1959); Sharma & Parton (2007)

a, b, c, d, e: model parameters, H: tree height (m), D: diameter at breast height (cm), H_0: dominant height (m), D_0: dominant diameter (cm), d_q: quadratic mean diameter (cm), G: basal area (m^2 / ha), N: tree numbers (trees/ha).

Root Mean Square Error:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(h_i - \hat{h}_i)^2}{n}}$$

Root Mean Square Error (%):

$$RMSE(\%) = \sqrt{\frac{\sum_{i=1}^{n}(h_i - \hat{h}_i)^2 / n}{\bar{h}_i}}\,100$$

Mean Absolute Error:

$$MAE = \frac{\sum_{i=1}^{n}|h_i - \hat{h}_i|}{n}$$

Mean Absolute Percentage Error:

$$MAPE = \frac{\sum|\hat{h}_i - h_i|}{\sum|h_i|}\,100$$

Akaike Information Criterion:

$$AIC = n\ln(RMSE) + 2p$$

where h_i, $\hat{h}_i$, $\bar{h}_i$ represent the observed, predicted and mean values of the dependent variable, n the number of observations, and p the number of model parameters, respectively.

In determining the best model, the relative ranking method proposed by Poudel and Cao (2013) was employed. For each statistical criterion, success rankings were assigned to the models, and their relative rankings were calculated. These values were then summed to determine the overall relative ranking for each respective model. The validity of the best model was tested using the testing data set (201 sample trees). For this purpose, the observed and predicted heights of the trees allocated as testing data were compared with the help of a Paired Samples *t*-test.

Results

Based on the sample plot data obtained from field works, individual tree and stand variables have been utilized in the development of *H-D* models. The relationships between stand variables and *H* are also illustrated in Fig. 3.

Model parameters for four base *H-D* models and five generalized *H-D* models were estimated using nonlinear regression analysis. The performance of all *H-D* models, for which all parameters were found to be statistically significant, was evaluated through relative ranking based on R^2, *RMSE*, *RMSE* (%), *MAE*, *MAPE*, and *AIC* statistical criteria, leading to the selection of the most successful nonlinear generalized *H-D* model (Tables 3–5).

Upon examination of the Table 5, it was found that the M1 and M6 equations were excluded from the relative ranking due to their statistically insignificant parameter values (Table 3). It is observed that the base *H-D* equation that best models the dataset is M3 (Ratkowsky & Reedy, 1986). The equation has an R^2 value of 0.601, with *RMSE* = 3.047, *RMSE*

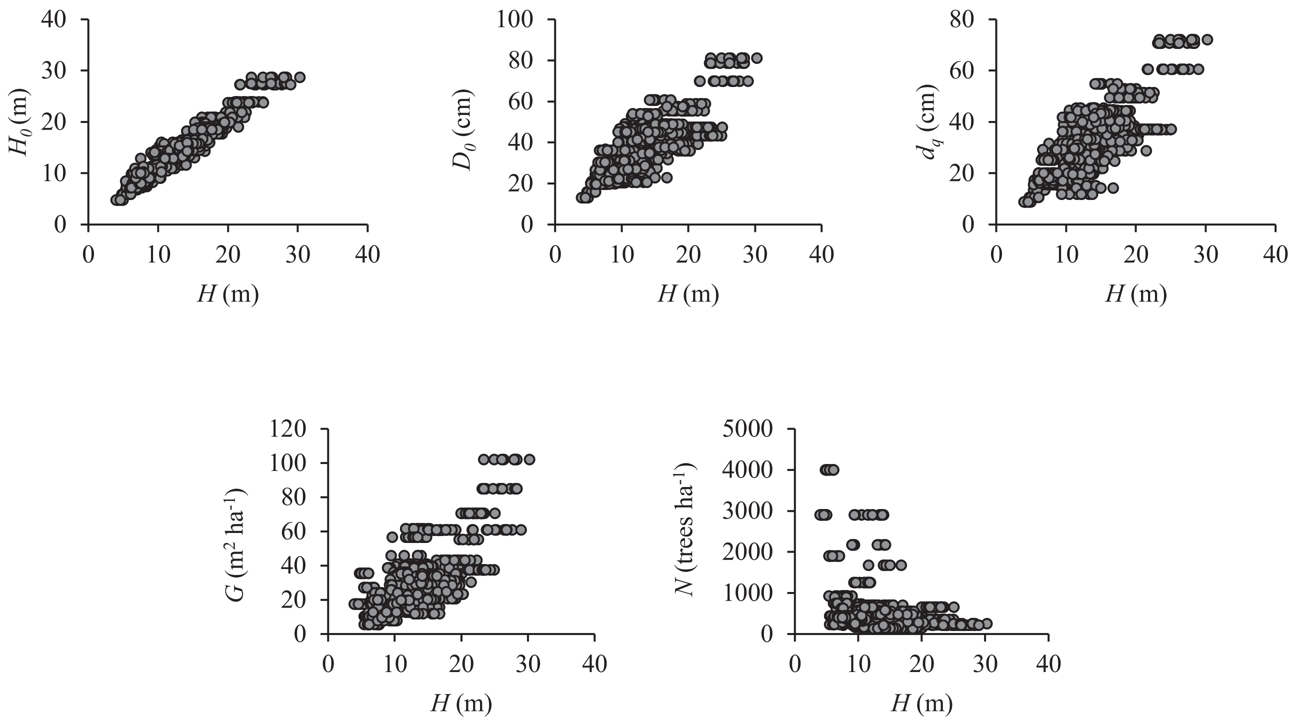

Fig. 3. Relationships between *H* and stand variables

Table 3. Parameters for *H-D* models

Model	Parameters	Parameters Values	Standard Deviation	*p*
		Base *H-D* Models		
M1	a	1.230	0.174	0.000
	b	27.554	1.300	0.000
	c	0.008[ns]	0.006	0.168
M2	a	41.762	14.683	0.005
	b	0.016	0.003	0.000
	c	0.893	0.086	0.000
M3	a	57.016	20.736	0.006
	b	0.010	0.001	0.000
	c	0.954	0.110	0.000
M4	a	39.040	10.644	0.000
	b	0.010	0.005	0.005
	c	0.878	0.092	0.000
		Generalized *H-D* Models		
M5	a	0.027	0.006	0.000
	b	1.948	0.230	0.000
M6	a	−3.569	0.310	0.000
	b	−0.280[ns]	0.205	0.171
	c	−0.010[ns]	0.009	0.266
M7	a	1.052	0.071	0.000
	b	1.286	0.907	0.001
	c	0.391	0.211	0.006
M8	a	0.745	0.293	0.011
	b	1.124	0.023	0.000
	c	0.068	0.010	0.000
	d	6.237	0.488	0.000
M9	a	0.772	0.036	0.000
	b	1.052	0.015	0.000
	c	0.059	0.009	0.000
	d	0.247	0.031	0.000
	e	0.689	0.597	0.005

[ns]$p>0.05$.

(%) = 23.155, *MAE* = 2.393, *MAPE* = 18.182, and *AIC* = 986.439.

It is observed that the generalized *H-D* equation that best models the dataset is M8 (Mirkovich, 1958), which has a relative ranking of 1.00. The selected model has an R^2 value of 0.941, with *RMSE* = 1.173, *RMSE* (%) = 8.914, *MAE* = 0.914, *MAPE* = 6.944, and *AIC* = 148.388. The second equation in the ranking of generalized models is M9, which has a relative ranking value of 1.02 and is the modified Richards (1959) equation by Sharma and Parton (2007).

An evaluation of Tables 4 and 5 reveals that the generalized *H-D* models yield superior predictions in comparison to the base *H-D* models. These models have demonstrated superior performance compared to the base *H-D* models in terms of coefficient of determination and statistical error metrics. Overall evaluation of all models indicates that the M8 model stands out prominently.

To evaluate the validity of the models, the heights of the trees in the testing data set were estimated using the developed *H-D* models. The observed and predicted heights of the trees in the testing data set were compared using a Paired Samples *t*-test. The results of the *t*-test indicated that the observed and predicted heights for the M2, M3, M4, M8, and M9 models exhibited no statistically significant difference ($p > 0.05$) (Table 6).

The predictions and residuals related to the developed *H-D* models were examined in Fig. 4 and 5. In the construction of the prediction graphs, observed and predicted heights calculated using *H-D* models (no statistically significant difference) were utilized. The residual graphs were constructed using

Table 4. Statistical criteria for *H-D* models

Model	R^2	RMSE	RMSE (%)	MAE	MAPE	AIC
M1	0.601	3.048	23.161	2.394	18.195	986.682
M2	0.600	3.048	23.163	2.394	18.196	986.757
M3	0.601	3.047	23.155	2.393	18.182	986.439
M4	0.601	3.048	23.167	2.395	18.198	986.883
M5	0.932	1.256	9.542	0.932	7.080	204.327
M6	0.932	1.256	9.544	0.931	7.077	206.467
M7	0.937	1.213	9.216	0.937	7.121	175.749
M8	0.941	1.173	8.914	0.914	6.944	148.388
M9	0.941	1.178	8.952	0.917	6.968	154.162

Table 6. *t*-test results for *H-D* models

Model	Mean	Standard deviation	*t*	*p*
		Base *H-D* Models		
M2	−0.22383	3.14286	−1.010	0.314
M3	−0.22637	3.14307	−1.021	0.308
M4	−0.22423	3.14289	−1.011	0.313
		Generalized *H-D* Models		
M5	−0.42731	1.22025	−4.965	0.000
M7	−0.20184	1.21505	−2.355	0.019
M8	−0.13328	1.17164	−1.613	0.108
M9	−0.13711	1.18211	−1.644	0.102

Table 5. Relative ranking of statistical criteria for *H-D* models

Model	Relative ranking							General ranking
	R^2	RMSE	RMSE (%)	MAE	MAPE	AIC	Total	
M2	7.00	7.00	7.00	7.00	7.00	7.00	41.99	7.00
M3	6.98	7.00	6.99	6.99	6.99	7.00	41.95	6.99
M4	6.98	7.00	7.00	7.00	7.00	7.00	41.98	7.00
M5	1.16	1.27	1.26	1.07	1.07	1.40	7.23	1.21
M7	1.07	1.13	1.13	1.09	1.09	1.20	6.71	1.12
M8	1.00	1.00	1.00	1.00	1.00	1.00	6.00	1.00
M9	1.00	1.02	1.02	1.01	1.01	1.04	6.10	1.02

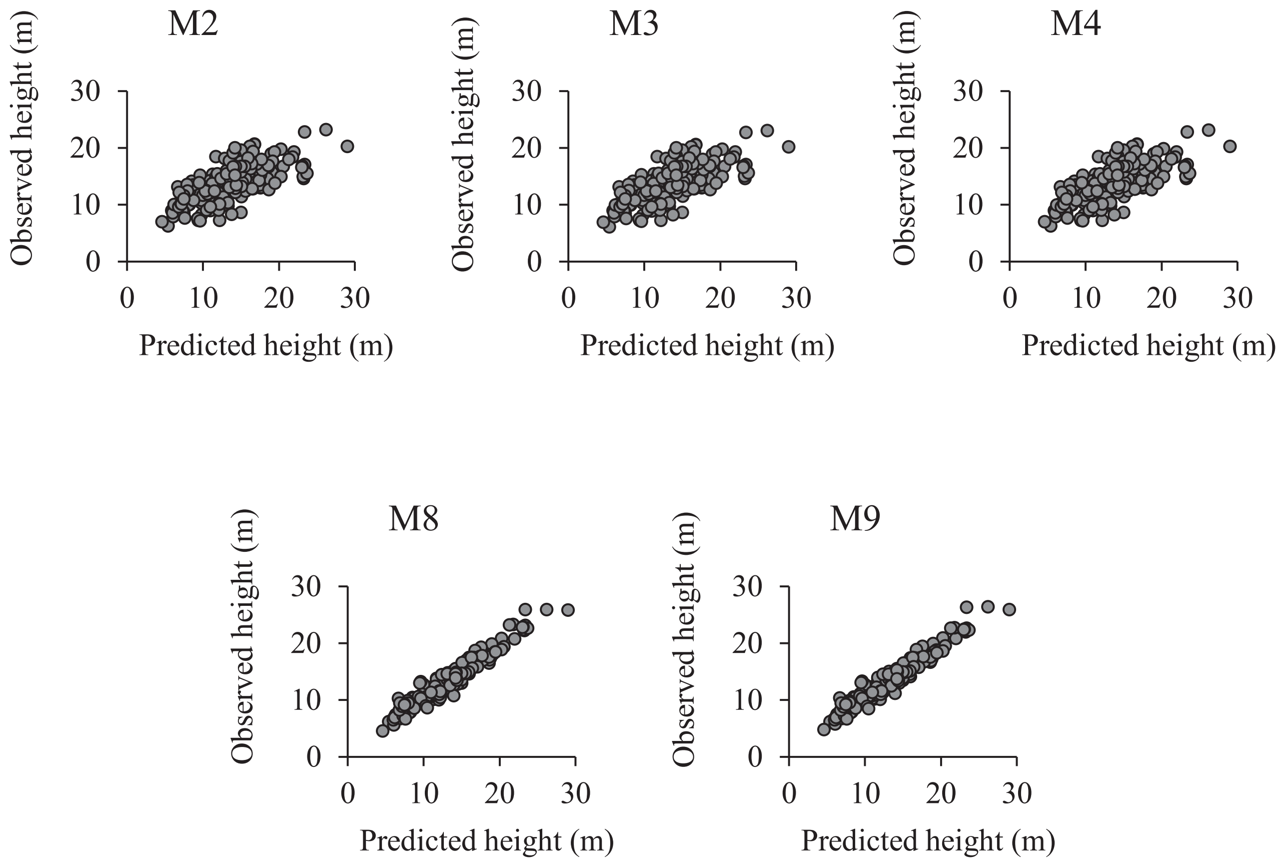

Fig. 4. Prediction of *H-D* models

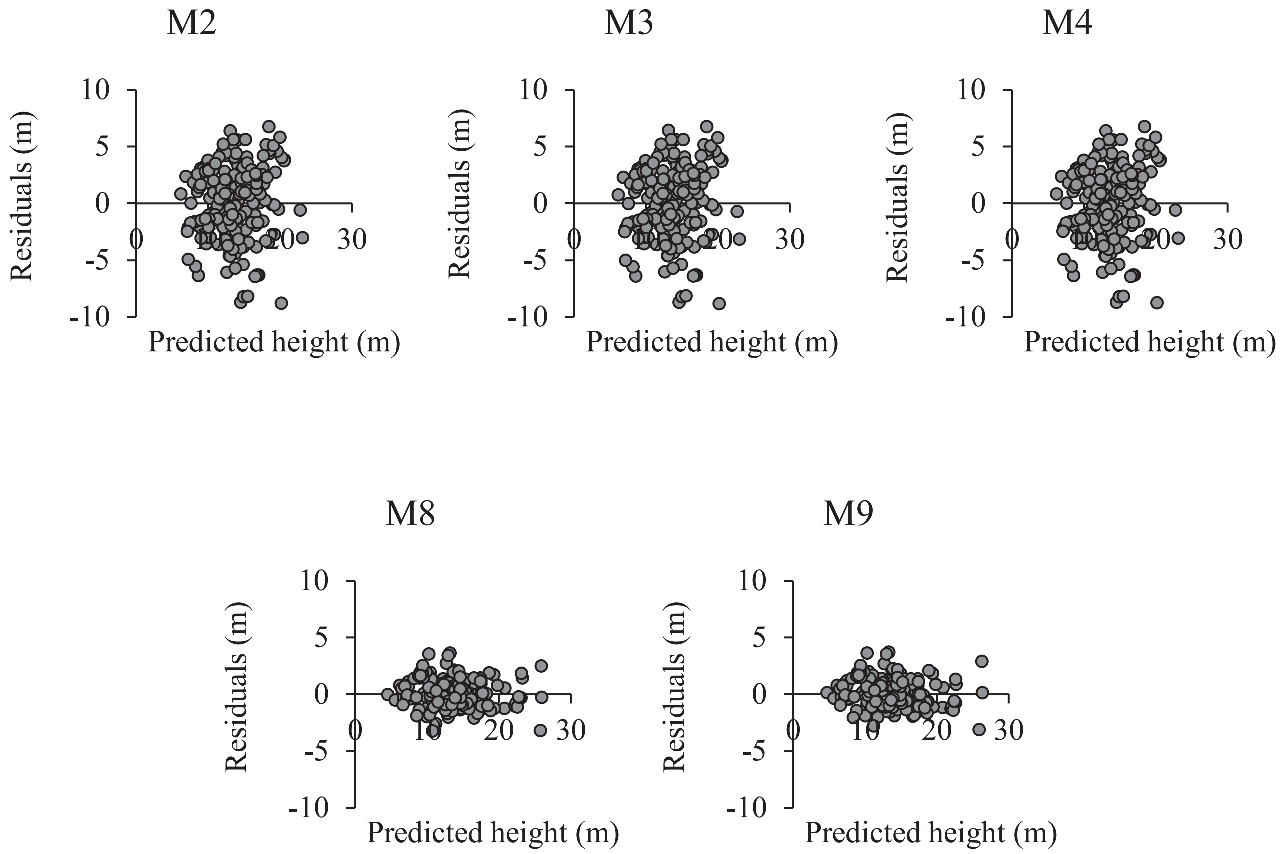

Fig. 5. Residuals of *H-D* models

the predicted heights obtained from *H-D* models and the corresponding residuals. Upon examining the figures, it was observed that the residuals associated with the predictions made by the *H-D* model exhibited a random distribution and did not display any trend. A closer examination of the residual plots (Fig. 5) reveals that the M8 model maintained low and unbiased residuals across a wide range of diameters. In particular, residuals were tightly clustered around zero for intermediate diameters, indicating high prediction accuracy in this range. Although slightly larger residuals were observed at the lower and upper extremes of the diameter distribution, no systematic bias was detected. This pattern suggests that the M8 model is robust across various tree sizes. These observations are consistent with the statistical criteria shown in Table 4, where M8 exhibited the lowest *RMSE* and highest R^2 values among the tested models.

As a result of statistical and graphical evaluations, M8 was selected as the best predictive *H-D* model. The model structure of M8 is outlined as follows:

$$H = 1.3 + (0.74540 + 1.12427 \times H_0 - 0.06842 \times d_q)\left(\exp\left(-\frac{6.23726}{D}\right)\right)$$

where H is tree height, D is diameter at breast height, H_0 is dominant height, d_q is quadratic mean diameter.

Discussion

In order to develop *H-D* models for Brutian pine stands in the Adana-Karaisalı region, parameter estimates were made for four base and five generalized *H-D* models. The statistical evaluations clearly demonstrated that generalized models performed better than the base models. Because *H-D* relationships vary from stand to stand, base models that rely solely on D performed less accurately.

In species such as *Pinus brutia*, which have a wide distribution and variable ecological conditions, the *H-D* relationship can vary significantly depending on stand variables and site conditions. Therefore, incorporating key stand condition indicators such as H_0 and d_q in the model substantially contributes to its predictive accuracy. The most predictive generalized *H-D* model, M8 (Mirkovich, 1958), incorporates D, H_0, and d_q as predictor variables in its structure. The inclusion of these variables in the model indicates that H_0 and d_q have significant effects on tree height. The H_0, which is an indicator of site productivity, is expected to have a significant effect on tree height when considered alongside diameter (Özçelik et al., 2018). This is because tree height tends to be higher in good site productivity conditions, while it is lower

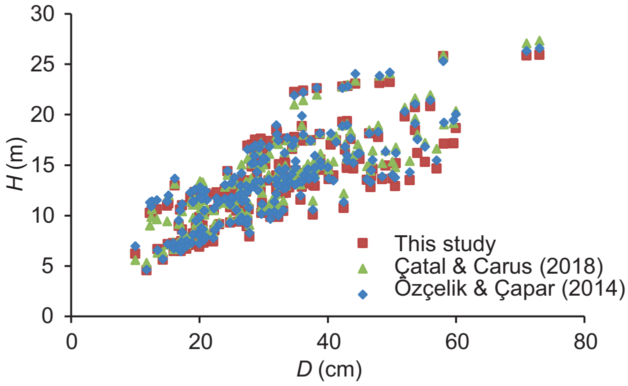

Fig. 6. Predictions for different *H-D* models for Brutian pine

in poor site conditions. Therefore, the inclusion of H_0 in the model improved model performance. This outcome corroborates findings from other forest ecosystems, where previous studies have shown that H_0 serves as a strong predictor in generalized *H-D* models, as it reflects site productivity and stand development characteristics that directly influence individual tree *H-D* relationships (Vargas-Larreta et al., 2009; Gómez-García et al., 2015; Özçelik et al., 2018; Santiago-García et al., 2020; Ciceu et al., 2023). Similarly, d_q also contributes to model performance by reflecting stand characteristics that influence height-diameter relationships. This is consistent with findings by Bronisz and Mehtätalo (2020), who similarly incorporated d_q as a predictor variable in their *H-D* models. These variables enhance the model's reliability and applicability in practical forestry operations.

A review of Fig. 4 and 5 shows that the residuals from the *H-D* model predictions were randomly distributed and exhibited no discernible trend. Moreover, predictions from the generalized models produced lower residuals. The analysis of the prediction and residual graphs confirms that generalized models outperform the base models. These assessments are consistent with the assumptions related to regression analysis, which state that residuals should exhibit a random distribution and lack any trend, serving as an important criterion for the model's success. Our findings align with those of Diamantopoulou et al. (2025), who reported that compared to basic Gompertz models, generalized models improved tree height prediction accuracy by reducing *RMSE*, attributing this improvement to the inclusion of stand-specific variables. This consistency across different species and study areas reinforces the general principle that incorporating stand-level variables significantly enhances the predictive performance of *H-D* models.

H-D models play a crucial role in forestry by serving as a foundation for inventory studies, forest management and forest practices. The studies conducted by Özçelik and Çapar (2014) and Çatal and Carus (2018) focused on the Brutian pine and were

Table 7. *t*-test results for different *H-D* models for Brutian pine

Model	Reference	Mean	Standard deviation	t	p
Mirkovich (1958)	This study	−0.13328	1.17164	−1.613	0.108
Sloboda et al. (1993)	Özçelik & Çapar (2014)	−0.37289	1.25276	−4.220	0.000
Sharma & Parton (2007)	Çatal & Carus (2018)	−0.49060	1.41521	−4.915	0.000

conducted in the Western Mediterranean region. In the study conducted by Carus and Çatal (2017), *H-D* models were developed for Brutian pine plantations. In another study of Özçelik et al. (2014), ecoregion-based *H-D* models were developed for Brutian pine, Crimean pine, and cedar. As no *H-D* model has been developed for the Brutian pine species in the study area, the results were compared with those of Özçelik and Çapar (2014) and Çatal and Carus (2018). These were conducted on pure and natural Brutian pine stands in the southwestern region of Türkiye (Fig. 6, Table 7). Due to the need for the stand mean height variable as an independent variable in the Cox IIa and Cox IIb models proposed by Çatal and Carus (2018), a comparison was made in this study with another successful model, the Sharma and Parton (2007) model.

An examination of Fig. 6 and Table 7 reveals that the predictions in this study differ from those of previous research. According to the results of the Paired Samples *t*-test conducted with the independent data set, it can be stated that the predicted heights of the two models, with the exception of the model M8, are statistically different from the observed heights. Particularly upon examining the graph, it is observed that while the height predictions are similar for the three models at lower diameter values, they diverge at higher diameter values. The models developed by Özçelik and Çapar (2014) and Çatal and Carus (2018) provide higher predictions compared to the model M8 developed within this study. The differences in model predictions are likely due to regional variation in the data used to develop the models, as evidenced by statistical and graphical analyses. This is due to the fact that *H-D* relationships vary among stands in different regions. As a result, it is essential to develop distinct *H-D* models for each region and species in order to obtain more reliable predictions.

Conclusion

The study area is characterized by a high concentration of Brutian pine stands and represents a significant distribution region for this species. Therefore, conducting this study will serve as an important foundation for tree and stand volume, biomass and carbon stock predictions for Brutian pine stands.

Within the scope of the study, four base and five generalized *H-D* models were selected. Based on assessment using statistical criteria, it is evident that the generalized models outperform the base models. Therefore, using the Mirkovich (1958) model, which is one of the generalized models, for height predictions in the relevant region will provide more unbiased predictions. H_0 and d_q are not always available in standard forest inventory practices. In such cases, we recommend the use of M3 (Ratkowsky & Reedy, 1986), which only requires D as a predictor. This model showed satisfactory performance in our analyses and provides a practical and reliable alternative when stand-level variables are unavailable.

As a result of the study, tree heights can be predicted either by measuring only D or by measuring D along with the H_0 and d_q. *H-D* models provide an important foundation for growth models and serve as a significant tool for estimating individual tree and stand volumes, as well as for predicting biomass and carbon stock. For this reason, studies related to *H-D* models for different tree species and in various regions, including those utilizing mixed-effects models and artificial neural networks to capture complex patterns and variability, are of great importance.

Authorship contribution

Fadime Sağlam: Supervision, Conceptualization, Methodology, Investigation, Validation, Visualization, Writing—review & editing.

Oytun Emre Sakici: Conceptualization, Methodology, Writing—review & editing.

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Conflict of interest

The authors declare that they have no conflict of interest.

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