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Upper stem diameter and volume predictions of Brutian pine (*Pinus brutia* Ten.) trees in Western Türkiye

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Abstract: Individual tree volume prediction is one of the most important components of growth and yield modeling systems. This is often accomplished by using taper equations because of their flexibility. In this study, stem taper models were developed to estimate upper stem diameters, merchantable volume, height, and total stem volume for natural Brutian pine (*Pinus brutia* Ten.) in the Akhisar region of western Türkiye. Performance of 11 commonly used taper models was analyzed using data collected from 159 Brutian pine trees representing a wide range of diameter and height classes. A second-order autoregressive error structure was specified to account for inherent autocorrelation in hierarchical data. The models of Jiang et al. (2005), Bi (2000), and Kozak (2004) had the best estimation performance for diameter, merchantable volume, height, and total stem volume estimations, respectively. If the diameter at 5.30 m is not known or cannot be estimated, either the models of Bi (2000) or Kozak (2004) are preferred for the merchantable volume, stem diameter, and total volume of Brutian pine trees in the study area. The best performing model (Jiang et al., 2005) was further improved using a non-linear mixed-effects modeling approach. Results showed that adding two random effects further improved model's estimation performance, decreased the error variance, and had better residual properties than the fixed effects model. The mixed-effects modeling approach is recommended for calibration when preliminary information for diameter measurements is available.

Keywords: diameter estimation, autocorrelation, nonlinear mixed-effects, relative ranking

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Introduction

The forest inventory data indicate that Brutian pine (*Pinus brutia* Ten.) in Türkiye covers an area of approximately 5.6 million hectares (approximately 25% of the forested areas of the country) with a current growing stock volume of nearly 270 million m^3 (GDF, 2015). The Brutian pine forests are distributed over a wide geographical area with different ecological characteristics and have a key role in important environmental issues such as the protection of soil and water resources, mitigating climate change impacts, and protecting biological diversity. This wide geographical distribution and the diverse ecological conditions need to be taken into account in the development of strategies for the management and planning of Brutian pine forests. However, research on the growth and yield of natural and pure Brutian pine in Türkiye has so far focused mostly on stands in southern Türkiye, even though the species is also widespread in the western part of the country.

Accurate estimates of tree and stand volumes and the distribution of this volume among different product categories is essential for any growth and yield modeling system. This information is critical to develop forest management plans (de-Miguel et al., 2012; Rodríguez et al., 2014), to estimate the forest products industry's future (de-Miguel et al., 2012), and to assess the effects of silvicultural interventions (Garber & Maguire, 2003). Furthermore, this information essential to calculate the amount of biomass and carbon sequestration with the help of appropriate biomass conversion factors (Castedo-Dorado et al., 2012), since the tree stem accounts for about 70% of all tree biomass (Poorter et al., 2012).

Besides total tree volume, one of the most crucial characteristics of tree that forest managers and researchers are interested in is the calculation of merchantable tree volume (Pancoast, 2018). Merchantable volume estimations are important for determining economic value based on changing market conditions, production planning, product price estimations, determining appropriate silvicultural interventions for stand establishment, energy production, and short or long-term economic analyses (Schröder et al., 2015; Pancoast, 2018).

However, origin (Poudel et al., 2020), tree and stand characteristics (Li & Weiskittel, 2010; Sharma & Zhang, 2004), crown variables (Liang et al., 2022), previous silvicultural interventions (Tasissa & Burkhart, 1998), and ecological conditions can all have an effect on stem form. Consequently, there may even be large differences between the stem forms of trees of the same diameter and height. Therefore, using local volume tables or volume equations for estimating tree stem volume by neglecting stem form, can lead to large volume estimation errors (Li & Weiskittel, 2010).

Due to their flexible structure, taper equations are a useful tool in growth modeling and forest inventory applications, both for estimating changes in stem form from the base to the top of the stem and for estimating whole stem or merchantable stem volume (Arias-Rodil et al., 2015; McTague & Weiskittel, 2020). Another important advantage of stem profile models for forestry applications is that they can be integrated into growth and yield models, allowing the estimation of product classes and quantities from different growing environments and different planning alternatives (de-Miguel et al., 2012). Hence, taper models have become one of the most widely used methods for tree volume prediction in recent years (Jiang et al., 2005; Sakici et al. 2008; McTague & Weiskittel, 2020; He et al., 2021; Wilms et al., 2024; Qadir & Poudel, 2025).

Stem profile models have been used for many purposes other than the above-mentioned common uses, such as estimating tree stem biomass (Parresol & Thomas, 1989), bark thickness (Yang & Radtke, 2022), bark volume (Yang & Qiao, 2024), and heartwood and sapwood diameter (Sun et al., 2024). Moreover, stem profile models can relate tree growth and timber quality models (Fonweban et al., 2012; Návar et al., 2013). Therefore, the development of stem profile models for different regions and different tree species is still one of the most important areas of study (Shahzad et al., 2020; He et al., 2022).

Two groups of stem diameter models have been widely and successfully used in forestry studies (Li et al., 2012). The first one is variable-exponent stem diameter models that a tree stem is thought to be composed of parts with neiloid, paraboloid, and taper stem shapes from the base to the top and the second group consists of segmented stem profile models that divide the tree stem into parts and define each part of the stem with different functions (Sharma, 2024). Detailed information about major advantages or disadvantages of the models in both two groups can be found with McTague & Weiskittel (2020).

Especially in the last three decades, studies have compared the estimation performance of different forms of stem taper models to estimate stem diameter, merchantable volume, and total volume. Li & Weiskittel (2010) compared the success of 10 different stem diameter models in estimating stem diameter and volume for main pine species in the Acadian Region of North America and found that the most successful results for stem diameter estimations were obtained with Kozak (2004) and Bi (2000) and for volume estimations with modified form of Clark et al. (1991). In a study conducted by Schröder et al. (2015) to develop a stem diameter model for slash pine in Brazil, six different stem diameter models were used, and the most successful results for diameter and volume estimations were obtained with the variable exponent stem diameter model of Kozak (1988). In a study conducted by Özçelik & Crecente-Campo (2016), ten different models in different forms were compared in terms of diameter, total volume, merchantable volume, and height estimations, and it was observed that the model developed by Clark et al. (1991) produced more successful results than the other models. In studies conducted by Hussain et al. (2020) using five different models for three different tree species and by Alkan & Özçelik (2020) using nine different stem diameter models for Taurus fir, it was observed that the model of Clark et al. (1991) produced the most accurate results for diameter, merchantable volume, and total volume estimations. Shahzad et al. (2020) compared eight stem diameter models in terms of diameter, merchantable and total volume estimations and found that the stem diameter model developed by Fang et al. (2000) presents the most successful results according to the evaluation criteria used. In another study, Shahzad et al. (2021) compared three different stem diameter models in terms of their success in stem taper estimation, and the most successful results were obtained with the model of Max & Burkhart (1976). Jiang et al. (2005) compared model forms developed by Max & Burkhart (1976), Clark et al. (1991) and as well as two reduced forms of Clark et al. (1991) to develop stem taper model for yellow poplar. The best results were produced by reduced form of Clark et al. (1991) for predicting upper stem diameter and volume. This reduced or modified form of Clark et al. (1991) will be referred to as Jiang et al. (2005)'s taper model in this study. Clark et al. (1991)'s and its reduced form (Jiang et al., 2005) use the diameter value at 5.30 m as an extra independent variable for upper stem diameter and volume predictions, unlike other taper models. The results of some studies (Li & Weiskittel, 2010; Özçelik & Crecente-Campo, 2016; Hussain et al., 2020) have shown that taper equations that include upper stem measurements generally outperform those that do not.

A limited number of studies have been performed in Türkiye to create merchantable volume equations for Brutian pine (Özçelik et al., 2016; Özçelik & Karaer, 2016). The existing tree volume prediction strategies used in Türkiye are not based on stem taper models and do not consider local ecological differences among regions. Currently, there are no known localized models for Brutian pine in western Türkiye, where local growing conditions are different than southern ecosystems. As indicated by Li et al. (2012), different growing conditions and stand attributes have a significant impact on stem characteristics of trees.

The ever-changing market conditions require accurate estimation of stem volume of a tree for different merchantable limits based on different upper stem diameter values. Unfortunately, this is not possible with the current Brutian pine tree volume tables in this region. By using upper stem diameter values from stem taper models, merchantable volume for different merchantable limits and stem volume of a tree would more accurately predict. Hence, in this study, we first evaluated the estimation performance of some well-known stem taper models for height, merchantable volume, diameter, and whole stem volume estimation for natural and pure Brutian pine stands in the Akhisar region of western Türkiye and then developed a tree-specific mixed-effects model based on the most successful model for upper stem diameter predictions.

Materials and methods

Materials

A total of 159 sample trees were selected from natural and pure Brutian pine (*Pinus brutia* Ten.) stands in the Akhisar region of western Türkiye (Fig. 1). Before the trees were felled, the diameter at breast height (D, 1.3 m above-ground) and after felled the heights (H) of sample trees were measured with precision of 0.1 cm and 5 cm for each tree, respectively. Stem diameter (d) values at 0.3 m, 2.3 m, and every 1 m thereafter were measured on felled each tree with a precision of 0.1 cm using electronic caliper. The trees were selected from dominant or co-dominant trees to represent all height and diameter classes within the study area. Since stem diameter models are not intended for deformed stems, trees with outliers such as trees with broken tops, forked stems, large knots, and some physical damage like growth deformations were excluded from the dataset. Descriptive statistics for the dataset are displayed in Table 1. Figure 2 shows scatter plot of relative diameter (d/D) against relative height (h/H) of the data used to develop stem profile models.

Table 1. Descriptive statistics of sample trees used for model fitting and validation

Data	Mean	SD	Minimum	Maximum
D (cm)	38.45	11.49	12.50	65.50
H (m)	20.43	4.86	9.70	29.60
d (cm)	23.95	12.70	1.50	70.20
h (m)	9.61	6.33	0.30	28.30
v (m^3)	0.841	0.71	0.02	4.24
V (m^3)	1.159	0.83	0.06	4.24

D – diameter outside bark at breast height (cm); H – total tree height (m); d – diameter outside bark to measurement point at h (cm); h – height above ground to measurement point (m); v and V – merchantable and total outside bark tree volumes (m^3), respectively; SD – standard deviation.

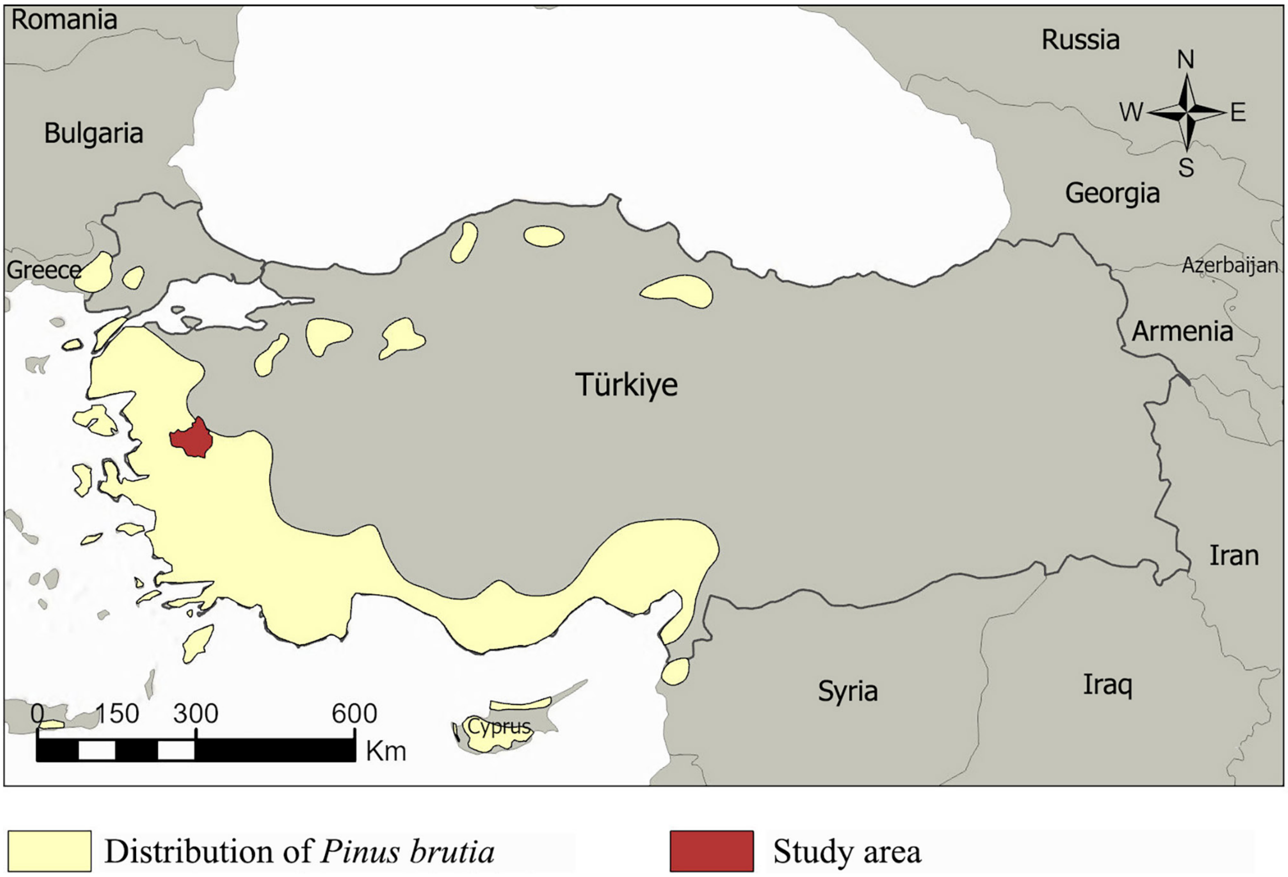

Fig. 1. Distribution of Brutian pine (*Pinus brutia* Ten.) across Türkiye and location of the Akhisar region, highlighting the study area

Kozak & Kozak (2003) stated that it is important to test the validity of models, but this requires the use of an independent data set. In case of insufficient or unavailable data, alternative methods such as cross-validation and data splitting can be used, but these methods rarely provide additional gains compared to model development using the entire data set. In this respect, since there is no independent dataset in this study, the complete dataset was used in the development, creation, and evaluation of the models.

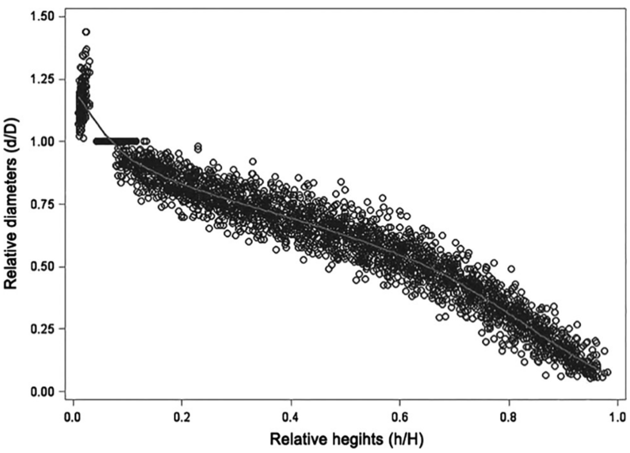

Fig. 2. Scatter plot of observed relative diameter (d/D) and relative height (h/H) with a loess smoothed curve for Brutian pine. Units of d and D are cm and unit of h and H are m, but the relative diameter and relative height are unitless

Methods

A total of 11 commonly used stem taper models were evaluated in this study. These models can be grouped as simple (Biging, 1984), segmented (Max & Burkhart, 1976; Fang et al., 2000; Jiang et al., 2005), and variable-exponent models (Riemer et al., 1995; Muhairwe, 1999; Bi, 2000; Lee et al., 2003; Kozak, 2004; Sharma & Zhang, 2004; Sharma & Parton, 2009). Table 2 presents detailed descriptions of these models. More detail information on these taper models can be found in the studies of Diéguez-Aranda et al. (2006), Li & Weiskittel (2010), and Wilms et al. (2024).

Diameter at 5.30 m is a required input variable for Jiang et al. (2005) taper equation, which is not used in other models. This can be obtained either by using the equation form presented in Jiang et al. (2005) or through actual field measurements. However, the measurement of diameter values at 5.30 m

Table 2. Mathematical representations of candidate stem taper models used in this study.

Model	Equation	
Max and Burkhart (1976)	$d = D[b_1(T-1) + b_2(T^2-1) + b_3(a_1-T)^2 I_1 + b_4(a_2-T)^2 I_2] \qquad I_i = \begin{cases} 1 & T \le a_i \\ 0 & T > a_i \end{cases}$	(1)
Biging (1984)	$d = D\left[b_1 + b_2 \ln\left(1 - T^{1/3}\right)\right]\left[1 - e^{\left(-b_1/b_2\right)}\right]$	(2)
Riemer et al. (1995)	$d = \frac{b_1 D}{1 - e^{b_3(1.3-H)}} + \left(\frac{D}{2} - b_1 D\right)\left[1 - \frac{1}{1 - e^{b_2(1.3-H)}}\right] + e^{-b_2 h}\left[\frac{\left(\frac{D}{2} - b_1 D\right) e^{1.3 b_2}}{1 - e^{b_2(1.3-H)}}\right] - e^{b_3 h}\left[\frac{b_1 D e^{-b_3 H}}{1 - e^{b_3(1.3-H)}}\right]$	(3)
Fang et al. (2000)	$d = c_1 \sqrt{H^{(k-b_1)/b_1}(1-T)^{(k-b)/b} \alpha_1^{I_1+I_2} \alpha_2^{I_2}}$ *where:* $k = \pi/40{,}000$, $T = h/H$, $\begin{cases} I_1 = 1 & \text{if } p_1 \le T \le p_2;\ 0 \text{ otherwise} \\ I_2 = 1 & \text{if } p_2 < T \le 1;\ 0 \text{ otherwise} \end{cases}$, $p_1 = h_1/H$ and $p_2 = h_2/H$ (p_1 and p_2 are inflection points at h_1 and h_2 are the heights from ground level), $b = b_1^{1-(I_1+I_2)} b_2^{I_1} b_3^{I_2},\ \alpha_1 = (1-p_1)^{(b_2-b_1)k/b_1 b_2},\ \alpha_2 = (1-p_2)^{(b_3-b_2)k/b_2 b_3}$, $r_0 = ((1-h_{st})/H)^{k/b_1},\ r_1 = (1-p_1)^{k/b_1},\ r_2 = (1-p_2)^{k/b_2},\ c_1 = \sqrt{\frac{a_0 D^{a_1} H^{a_2 - k/b_1}}{b_1(r_0-r_1) + b_2(r_1-\alpha_1 r_2) + b_3 \alpha_1 r_2}}$,	(4)
Bi (2000)	$d = D\left[\log\sin\left(\frac{\pi}{2}T\right) \Big/ \log\sin\left(\frac{\pi}{2}\frac{1.3}{H}\right)\right]^{b_1 + b_2 \sin((\pi/2)T) + b_3 \cos((3\pi/2)T) + b_4 \sin((\pi/2)T)/T + b_5 D + b_6 T\sqrt{D} + b_7 T\sqrt{H}}$	(5)
Lee et al. (2003)	$d = b_1 D^{b_2}(1-T)^{b_3 T^2 + b_4 T + b_5}$	(6)
Kozak (2004)	$d = a_0 D^{a_1} H^{a_2}\left[\frac{1 - T^{1/3}}{1 - p^{1/3}}\right]^{\left[b_1 T^4 + b_2(1/e^{D/H}) + b_3\left(\frac{1-T^{1/3}}{1-p^{1/3}}\right)^{0.1} + b_4\left(\frac{1}{D}\right) + b_5 H^{1-\left(\frac{h}{H}\right)^{1/3}} + b_6\left(\frac{1-T^{1/3}}{1-p^{1/3}}\right)\right]}$	(7)
Sharma and Zhang (2004)	$d = D\left(b_1\left(\frac{H-h}{H-1.3}\right)\left(\frac{h}{1.3}\right)^{2-(b_2+b_3T+b_4T^2)}\right)^{0.5}$	(8)
Sharma and Parton (2009)	$d = D\left(b_1\left(\frac{H-h}{H-1.3}\right)\left(\frac{h}{1.3}\right)^{(b_2+b_3T+b_4T^2)}\right)$	(9)
Muhairwe (1999)	$d = a_0 D^{a_1} a_2^D [1 - \sqrt{T}]^{\left(b_1 T^2 + (b_2/T) + b_3 D + b_4 H + b_5(D/H)\right)}$	(10)
Jiang et al. (2005)	$d = \left\{\begin{matrix} I_S\left[D^2\left(1 + \frac{(1-T)^{b_1} - (1-1.30/H)^{b_1}}{1-(1-1.30/H)^{b_1}}\right)\right] + \\ I_B\left[D^2 - \frac{(D^2-F^2)(1-1.30/H)^{b_2} - (1-T)^{b_2}}{(1-1.30/H)^{b_2} - (1-5.30/H)^{b_2}}\right] + \\ I_T\left[F^2\left(b_4\left(\frac{h-5.30}{H-5.30} - 1\right)^2 + I_M\left(\frac{1-b_4}{b_3^2}\right)\left(b_3 - \frac{h-5.30}{H-5.30}\right)^2\right)\right] \end{matrix}\right\}^{0.5}$ Jiang et al. (2005) proposed the following equation to estimate diameter at 5.30 m: $F = D(p_1 + p_2(5.30/H)^2)$.	(11)

$T = h/H$; h_{st} is stump diameter height; a_i, b_i and p_i are the parameters to be estimated; all other variables as previously defined.

is generally not made during routine forest inventories. In this study, diameters at 5.30 m were obtained through linear interpolation and actual field measurements. The model fitted using interpolated 5.30 m diameter will be referred to as Jiang et al. (2005)-I and the model fitted with observed diameter at 5.30 m will be referred to as Jiang et al. (2005)-II for the remainder of this article.

Statistical evaluation

When using the ordinary least squares (*OLS*) method to develop stem diameter models, two main problems, multicollinearity and autocorrelation, are frequently observed (Kozak, 1997). Therefore, it is emphasized that appropriate statistical approaches should be selected to avoid the autocorrelation problem and reduce the multicollinearity problem while developing the taper equations (Crecente-Campo et al., 2009). To assess the existence of multicollinearity among the variables in the model structure, Condition Number (CN) was used. According to Belsey (1991), if the condition number is in the range of 5 to 10, there is no multicollinearity problem; if it is in the range of 30 to 100, there may be a problem with multicollinearity, but if it is in the range of 1000 to

3000, there may be a severe multicollinearity problem. As the sample tree data collected contains a large number of observations from each tree, it is expected that the observations in each tree are spatially correlated with each other, not consistent with the assumption of independence of error terms. Therefore, the inherent autocorrelation in the hierarchical data structure was accounted for by using a second-order continuous autoregressive error structure (CAR(2)), which takes into account the distance between diameter measurements and visually examined residual plots to determine if the autocorrelation issue had been resolved (Diéguez-Aranda et al., 2006).

Model comparison

The accuracy of stem diameter, merchantable volume, and volume predictions were evaluated using numerical and graphical analysis of residuals. Four different evaluation criteria, including coefficient of determination (R^2; Equation 12), root mean square error (RMSE; Equation 13), mean absolute difference (MAD; Equation 14), and Akaike's Information Criterion (Akaike 1974; AIC; Equation 15). The best model was selected using the relative ranking method suggested by Poudel & Cao (2013).

$$R^2 = 1 - \frac{\sum_{i=1}^{i=n}\left(Y_i - \hat{Y}_i\right)^2}{\sum_{i=1}^{i=n}(Y_i - \bar{Y}_i)^2} \quad (12)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{i=n}\left(Y_i - \hat{Y}_i\right)^2}{n-p}} \quad (13)$$

$$MAD = \frac{\sum_{i=1}^{i=n}\left|Y_i - \hat{Y}_i\right|}{n} \quad (14)$$

$$AIC = nlog\left(\sum_{i=1}^{i=n}\left(Y_i - \hat{Y}_i\right)^2 /n\right) + 2p \quad (15)$$

where $\bar{Y}_i$, Y_i, and $\hat{Y}_i$ are mean of observed, estimated and observed values, respectively; n is the number of observations used to develop the model; and is the number of parameters in the model.

Results

When the stem taper models used in this study were fitted without considering the presence of autocorrelation, the residuals of the models generally showed a similar trend and the resulting error values and the lag error showed a linear correlation. Analyzing the results obtained for Jiang et al. (2005) in Figure 3, it is seen that when a first order autoregressive error structure is added to the model (CAR(1)), the autocorrelation problem is partially eliminated (Fig. 3, Row 2, Column 2) but when a second order autoregressive error structure is added (CAR(2)),

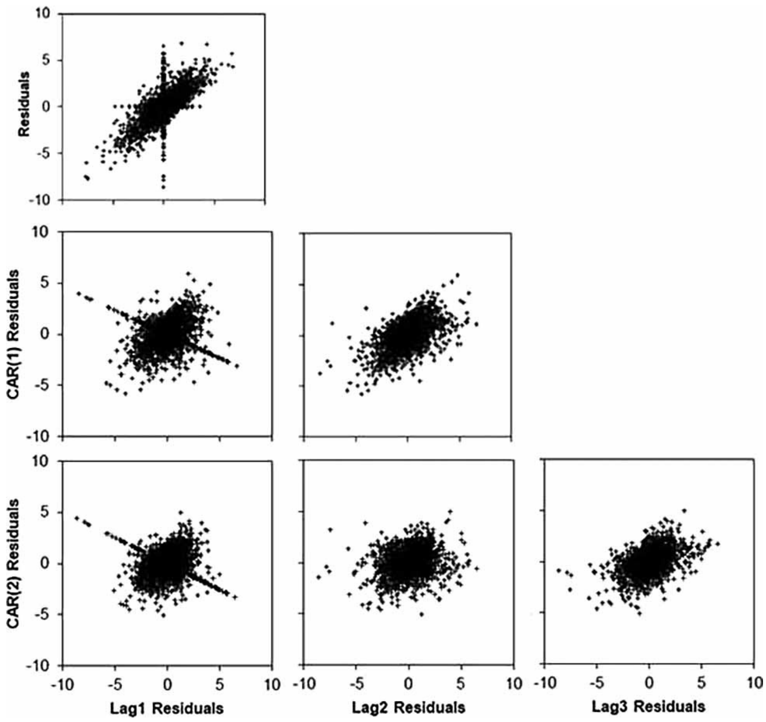

Fig. 3. An example, of *d* residuals plotted against: Lag1-residuals, Lag2-residuals, and Lag3-residuals for the model of Jiang et al. (2005) fitted without considering the autocorrelation parameters (first row), and with continuous autoregressive error structures of first and second order (second and third rows, respectively)

Table 3. Parameter estimates with their associated standard errors (SE) in parentheses for the analyzed models.

Models	a_0	a_1	a_2	b_1	b_2	b_3	b_4	b_5	b_6	b_7	p_1	p_2
Max and Burkhart (1976)		0.7889 (0.0125)	0.1576 (0.0045)	−4.4859 (0.2375)	2.2201 (0.1278)	−2.2339 (0.1225)	21.5772 (1.1057)					
Biging (1984)				1.2459 (0.0037)	0.4795 (0.0049)							
Riemer et al. (1995)				0.4397 (0.0030)	0.6898 (0.0243)	0.0699 (0.0021)						
Fang et al. (2000)	0.000054 (2.79×10^{-6})	1.9909 (0.0204)	0.8434 (0.0233)	0.000014 (1.97×10^{-7})	0.000035 (2.41×10^{-7})	0.000027 (4.09×10^{-7})					0.1101 (0,0020)	0.7034 (0.0076)
Bi (2000)				0.2892 (0.0718)	−0.0900 (0.0374)	0.0618 (0.0083)	0.0298* (0.0454)	−0.0031 (0.0003)	0.0468 (0.0069)	0.0005* (0.0070)		
Lee et al. (2003)				1.5974 (0.0411)	0.9198 (0.0069)	3.1875 (0.0988)	−4.5004 (0.1210)	2.3766 (0.0377)				
Kozak (2004)	1.0938 (0.0355)	0.9578 (0.0104)	0.0208* (0.0138)	0.4989 (0.0196)	0.3579 (0.0769)	0.2822 (0.0142)	1.5272 (0.5960)	0.0012* (0.0043)	0.1606 (0.0361)			
Sharma and Zhang (2004)				0.9527 (0.0060)	2.1759 (0.0040)	−0.4707 (0.0360)	0.7509 (0.0431)					
Sharma and Parton (2009)				0.9900 (0.0030)	−0.0649 (0.0019)	0.2924 (0.0174)	−0.0182* (0.0212)					
Muhairwe (1999)	774222.3 (243893)	0.8822 (0.0081)	−13.0827 (0.8065)	11.1743 (1.5251)	25.8894 (0.6135)	−0.3108* (0.9267)	−0.0046 (0.0013)	−0.3191 (0.0327)				
Jiang et al. (2005)-I				78.7927 (1.5703)	3.8335 (0.4294)	0.7523 (0.0131)	2.2068 (0.0627)					
Jiang et al. (2005)-II				77.5560 (1.4462)	6.4987 (0.4130)	0.7363 (0.0097)	2.2343 (0.0447					

An asterisk (*) indicates non-significant parameters at $p< 0.05$. Jiang et al. (2005)-I: upper stem diameters at 5.30 m were obtained by linear interpolation. Jiang et al. (2005)-II: upper stem diameters at 5.30 m were obtained by actual field measurements.

the autocorrelation problem is almost eliminated (Fig. 3, Row 3, Column 3).

The parameter estimations and their standard error values for the used models are given in Table 3. Among the analyzed models, the parameters and of Bi (2000), and of Kozak (2004), of Sharma & Parton (2009), and of Muhairwe (1999) were found insignificant at the 0.05 level of significance. All other parameters of the analyzed models were found to be significant at the $p<0.0001$ level. The models with statistically insignificant parameters were solved again after removing these parameters from the model, but there was no change or improvement in the models' estimation performance. Therefore, the models were used in their original form.

Based on the Condition Number for the models in Table 4, it is seen that a significant portion of the models have a moderate multicollinearity problem. The models of Biging (1984), Jiang et al. (2005) and Riemer et al. (1995) do not show multicollinearity problems in terms of condition numbers; the models of Muhairwe (1999), Bi (2000) and Max & Burkhart (1976) show multicollinearity problems and the remaining models generally have moderate multicollinearity problems (CN: 14-71).

Table 4 presents the goodness-of-fit statistics values for the performance of the analyzed taper models in merchantable volume, height, diameter, and total stem volume estimations. As seen in Table 4, more than 94%, 98%, 96%, and 96% of the total variance in height, diameter, merchantable volume and total stem volume estimations, respectively, can be explained by the taper models. The RMSE values ranged between 1.34–1.60 cm, 1.27–1.41 m, 0.05–0.12 m^3, and 0.07–0.15 m^3 for diameter, height, merchantable volume, and total stem volume estimations, respectively, while MAD values ranged between 0.98–1.19 cm, 0.92–1.03 m, 0.03–0.08 m^3, and 0.04–0.096 m^3 for the same variables, respectively. Table 5 presents the relative ranking of the models for the four criterion values used for diameter, height, merchantable volume, and total volume estimations. According to the relative ranking results based on R^2, RMSE, MAD, and AIC values produced by the models in Table 5, the models with the best estimation performance are those of Jiang et al. (2005), Bi (2000), and Kozak (2004), while the models with the lowest estimation performance are those of Biging (1984), Riemer et al. (1995), and Lee et al. (2003). Results also show that using interpolation method (Jiang et al., 2005) to predict the upper stem diameter at 5.30 m largely increased the bias (over 17%) for estimating *d* (outside bark diameter) a, compared with actual field measurements to obtain upper stem diameter values. The performance rank of volume prediction among the eleven taper models showed a slightly different trend than *d* and height predictions. Jiang et

Table 4. Goodness-of-fit statistics and condition numbers for the analyzed models

Models	Diameter				Height*				Merchantable volume				Total volume				CN
	R^2	RMSE	AIC	MAD	R^2	RMSE	AIC	MAD	R^2	RMSE	AIC	MAD	R^2	RMSE	AIC	MAD	
Max and Burkhart (1976)	0.9875	1.4225	2056	1.0767	0.9517	1.3395	1612	0.9951	0.9773	0.1076	−12993	0.0655	0.9690	0.1401	−615	0.0917	190
Biging (1984)	0.9852	1.5459	2535	1.1584	0.9529	1.3215	1534	0.9916	0.9698	0.1242	−12104	0.0763	0.9658	0.1453	−607	0.0956	4
Riemer et al. (1995)	0.9870	1.4473	2154	1.1110	0.9491	1.3751	1753	1.0110	0.9773	0.1076	−12934	0.0631	0.9671	0.1429	−612	0.0934	7
Fang et al. (2000)	0.9881	1.3908	1927	1.0626	0.9538	1.3100	1492	0.9602	0.9786	0.1046	−13097	0.0589	0.9711	0.1346	−630	0.0835	55
Bi (2000)	0.9889	1.3375	1699	1.0265	0.9561	1.2778	1356	0.9255	0.9806	0.0996	−13379	0.0581	0.9728	0.1317	−634	0.0831	183
Lee et al. (2003)	0.9874	1.4274	2076	1.0952	0.9467	1.4071	1883	1.0274	0.9759	0.1109	−12759	0.0668	0.9680	0.1420	−612	0.0921	48
Kozak (2004)	0.9889	1.3412	1717	1.0295	0.9568	1.2681	1314	0.9248	0.9793	0.1030	−13185	0.0583	0.9722	0.1332	−630	0.0823	71
Sharma and Zhang (2004)	0.9872	1.4406	2127	1.1063	0.9528	1.3237	1545	1.0051	0.9785	0.1048	−13088	0.0595	0.9709	0.1348	−629	0.0834	14
Sharma and Parton (2009)	0.9881	1.3882	1919	1.0588	0.9553	1.2887	1398	0.9459	0.9790	0.1034	−13164	0.0599	0.9711	0.1345	−630	0.0862	14
Muhairwe (1999)	0.9884	1.3700	1838	1.0540	0.9521	1.3356	1600	0.9730	0.9788	0.1040	−13128	0.0603	0.9721	0.1330	−632	0.0832	648
Jiang et al. (2005)-I	0.9842	1.5966	2724	1.1872	0.9520	1.3356	1594	0.9709	0.9763	0.1100	−12807	0.0662	0.9680	0.1415	−598	0.0936	4
Jiang et al. (2005)-II	0.9885	1.3627	1805	0.9787	0.9538	1.3092	1484	0.9487	0.9952	0.0486	−17571	0.0282	0.9926	0.0677	−849	0.0407	4

* The bisection method used to give an explicit mathematical solution of height (h) to a given diameter (d). A numerical solution was obtained through iterations using this method. CN: Condition number. All other variables as previously defined.

Table 5. Relative rankings based on goodness-of-fit statistics for the analyzed models

Models	Diameter				Height				Merchantable volume				Total volume				Sum of rank	Relative rank
	r^2	RMSE	AIC	MAD	R^2	RMSE	AIC	MAD	R^2	RMSE	AIC	MAD	R^2	RMSE	AIC	MAD		
Max and Burkhart (1976)	4.2766	4.6086	4.8312	6.1703	6.5545	6.6504	6.7610	8.5370	8.7520	9.5847	10.211	9.5301	10.686	11.262	11.255	11.218	38.3661	10.1889
Biging (1984)	9.6596	9.8475	9.9717	10.480	5.2475	5.2259	5.2531	8.1618	**12.000**	**12.000**	**12.000**	**12.000**	**12.000**	**12.000**	11.605	**12.000**	41.9721	11.2484
Riemer et al. (1995)	5.4468	5.6615	5.8829	7.9799	9.3861	9.4676	9.4868	10.241	8.7520	9.5847	10.330	8.9813	11.466	11.659	11.386	11.559	41.8855	11.2229
Fang et al. (2000)	2.8723	3.2628	3.4468	5.4264	4.2673	4.3158	4.4411	4.7953	8.1890	9.1481	10.002	8.0208	9.8246	10.483	10.597	9.5756	33.1320	8.6510
Bi (2000)	**1.0000**	**1.0000**	**1.0000**	3.5218	1.7624	1.7676	1.8120	1.0750	7.3228	8.4206	9.4346	7.8378	9.1269	10.072	10.422	9.4954	28.4472	7.2745
Lee et al. (2003)	4.5106	4.8167	5.0459	7.1463	**12.000**	**12.000**	**12.000**	**12.000**	9.3583	10.064	10.682	9.8274	11.097	11.53	11.386	11.298	44.5302	**12.0000**
Kozak (2004)	**1.0000**	1.1571	1.1932	3.6801	1.0000	1.0000	1.0000	1.0000	7.8858	8.9153	9.8249	7.8836	9.3731	10.284	10.597	9.3352	28.3596	7.2488
Sharma and Zhang (2004)	4.9787	5.3771	5.5932	7.7319	5.3564	5.4000	5.4657	9.6092	8.2323	9.1772	10.020	8.1580	9.9067	10.511	10.641	9.5556	35.9639	9.4830
Sharma and Parton (2001)	2.8723	3.1525	3.3610	5.2259	2.6337	2.6302	2.6239	3.2622	8.0157	8.9735	9.8672	8.2495	9.8246	10.469	10.597	10.116	31.4149	8.1465
Muhairwe (1999)	2.1702	2.3798	2.4917	4.9727	6.1188	6.3417	6.5290	6.1676	8.1024	9.0608	9.9396	8.3410	9.4142	10.256	10.510	9.5155	34.6835	9.1068
Jiang et al. (2005)-I	**12.000**	**12.000**	**12.000**	**12.000**	6.2277	6.3417	6.4130	5.9425	9.1850	9.9339	10.585	9.6902	11.097	11.461	**12.000**	11.599	30.0908	7.7574
Jiang et al. (2005)-II	1.9362	2.0699	2.1376	1.0000	4.2673	4.2525	4.2865	3.5624	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	7.0922	**1.0000**

al. (2005) model with observed upper stem diameter at 5.30 m produced significantly lower RMSE and MAD. The poorest performing taper model for predicting stem volume was the Biging (1984). The best performing taper equation of Jiang et al. (2005) reduced the stem merchantable volume and stem total volume RMSE by 60% and 53% when compared to the poorest performing taper model of Biging (1984) for Brutian pine, respectively. When observed upper stem diameter measurements were not available, the Bi (2000) equation reduced the stem merchantable and stem total volumes RMSE by 12% and 9% when compared to the Biging (1984) taper model.

Figure 4 presents the radar plot for the three most successful and three least successful models based on the relative ranking values of the four criteria values. As can be seen from this figure, the best model, Jiang et al. (2005), is located in the center, while the most unsuccessful models, those of Biging (1984), Riemer et al. (1995), and Lee et al. (2003), are located on the outside.

The rank of each variable calculated for R^2, RMSE, AIC, and MAD. The relative rank was based on the sum of the ranks over all evaluation statistics. A bold, italic number denotes relative rank of the best taper model and a bold, underlined number denotes the worst taper model among the used models.

Figures 5, 6, 7, and 8 present the distribution and variation of the errors for diameter, height, merchantable volume, and total stem volume estimations by relative diameter, relative height, and diameter classes, respectively. In general, the error distributions of the models along the tree stem show similar trends for all models, and it can be said that

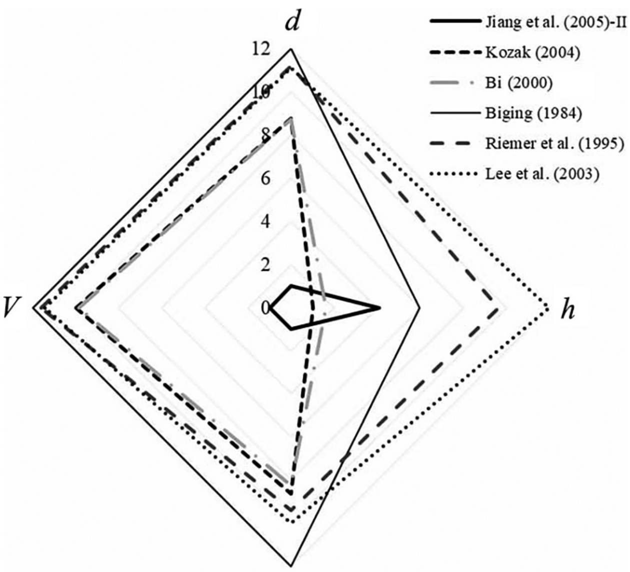

Fig. 4. The three most and least successful models in estimating stem diameter, height, merchantable volume, and total volume

they have similar error distributions for the same relative height classes, relative diameter classes, and diameter classes. More successful estimations were produced in 15–55% of the stem than in other relative diameter classes for diameter estimations of all models (Fig. 5).

All models produced larger standard errors for height estimations above 45% of total tree height (Fig. 6). With the exception of Lee et al. (2003), all models tended to produce higher magnitude values for relative diameter classes below 20%.

Figure 7 and Figure 8 show the distribution of errors in the estimation of merchantable volume and total stem volume by diameter class; all models showed larger estimation errors for larger diameter classes than for smaller diameter classes. However, Jiang et al. (2005) found more successful estimations for larger diameter classes than the other analyzed models. All models overestimated the actual volume values for trees 32 cm and above, except for the 52–57 cm diameter class to estimate total volume. According to the four criteria used in the study and graphical evaluations, the most successful results for diameter, merchantable length, merchantable volume, and total volume estimations were obtained with the model of Jiang et al. (2005).

In the next stage of the study, the stem taper model developed by Jiang et al. (2005), which is a reduced form of the model of Clark et al. (1991), was used. Jiang et al. (2005) was transformed into a nonlinear mixed-effect model by adding random effects to some of the fixed-effect parameters of the model in order to obtain tree-specific parameters for each tree. For this purpose, the model of Jiang et al.

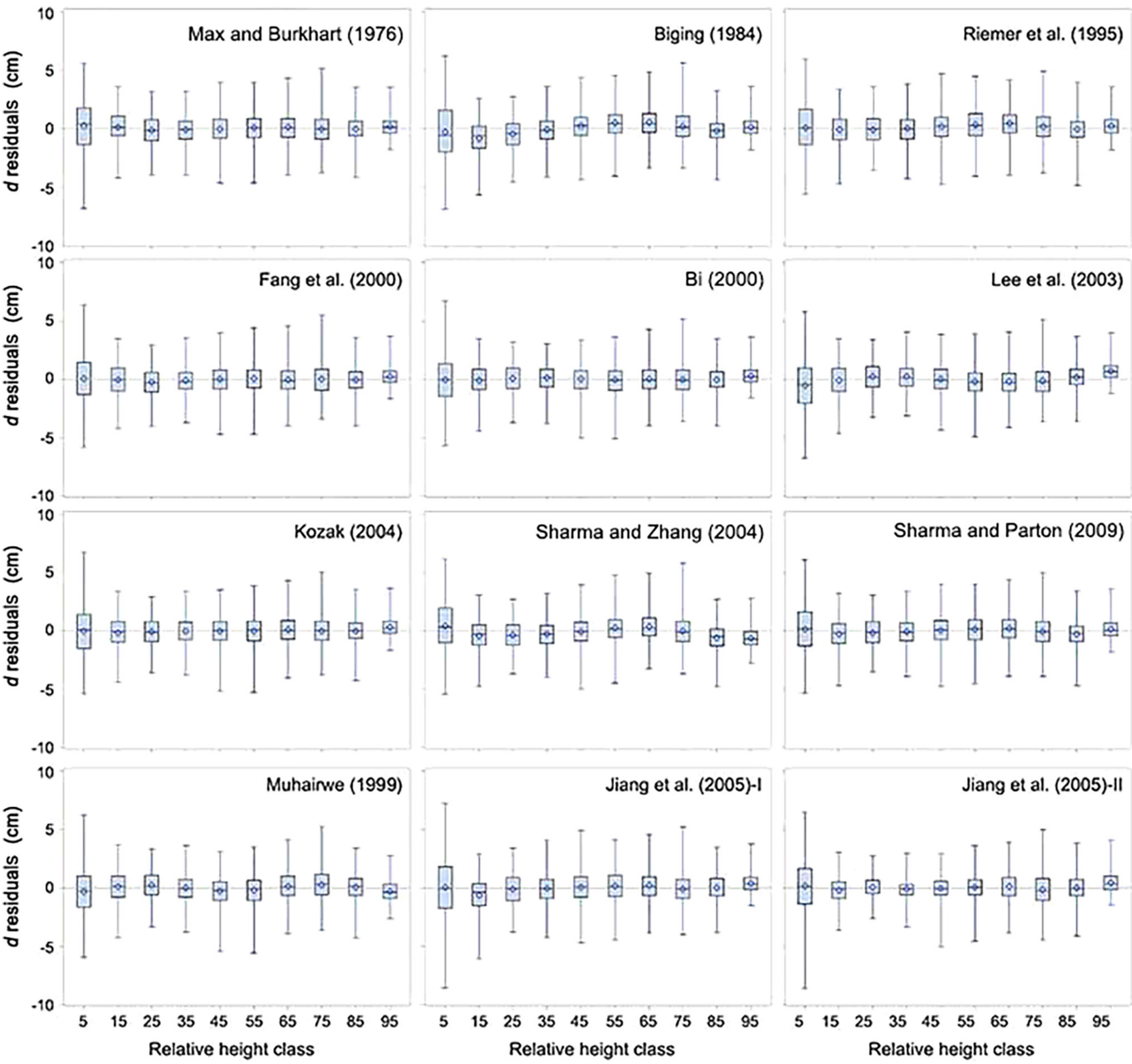

Fig. 5. Box plots of *d* residuals versus relative height classes (in percent) for the models

(2005) was re-solved for possible combinations of fixed-effect and random-effect parameters. In line with the results of similar studies (Hussain et al., 2021a; Hussain et al., 2021b), the solution was not possible for mixed-effects models with more than two random-effects parameters. The criterion values obtained for different random parameter combinations are given in Table 6. As can be seen from Table 6, a total of 10 mixed-effects models with only one or two parameters with random effects were produced within the scope of the study.

Again, as can be seen in Table 6, according to AIC and BIC values, the most successful random effect parameter combinations are b_2 and b_4. The criterion values of this mixed-effects model are significantly better than the fixed-effects model. The final mixed-effects form of the model of Jiang et al. (2005) is as follows.

$$d = \left\{ \begin{array}{c} I_s\left[D^2\left(1+\frac{(1-T)^{b_1}-(1-1.30/H)^{b_1}}{1-(1-1.30/H)^{b_1}}\right)\right]+ \\ I_B\left[D^2-\frac{(D^2-F^2)(1-1.30/H)^{(b_2+u_1)}-(1-T)^{(b_2+u_1)}}{(1-1.30/H)^{(b_2+u_1)}-(1-5.30/H)^{(b_2+u_1)}}\right]+ \\ I_T.\left[F^2\left((b_4+u_2)\left(\frac{h-5.30}{H-5.30}-1\right)^2+I_M\left(\frac{1-(b_4+u_2)}{b_3^2}\right)\left(b_3-\frac{h-5.30}{H-5.30}\right)^2\right)\right] \end{array} \right\}^{0.5}$$

Table 7 presents the fixed-effect parameter estimations and variance components of the mixed-effects model of Jiang et al. (2005).

Figure 9 presents the error distribution produced by the fixed-effect (Fig. 9A) and mixed-effect (Fig. 9B) models in diameter estimation for relative height classes. The mixed-effect model has a much more homogeneous error distribution than the fixed-effect model for relative height classes.

Table 8 presents the goodness-of-fit statistics values for the performance of the Jiang et al. (2005)

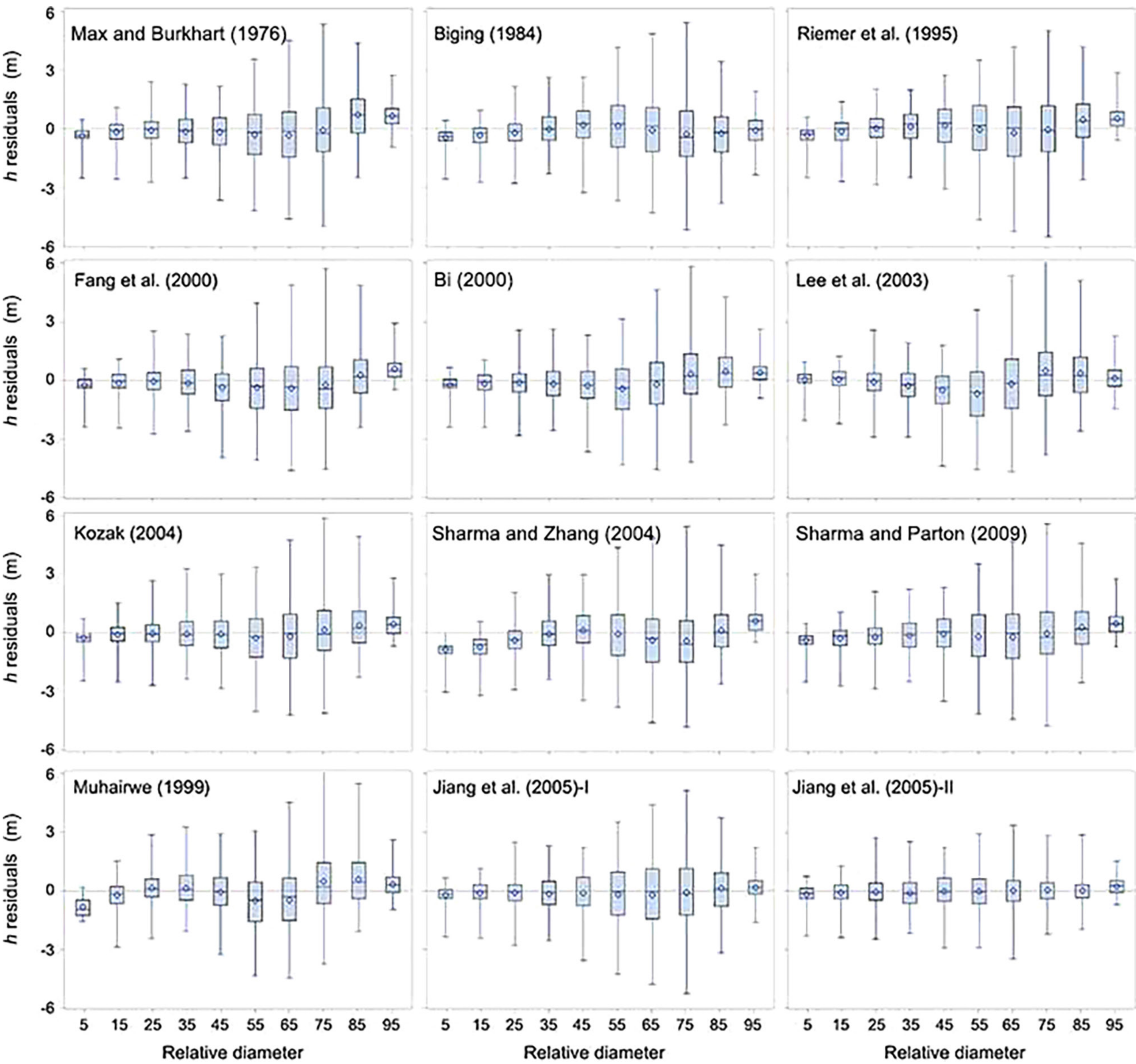

Fig. 6. Box plots of h residuals versus relative diameter classes (in percent) for the models

Table 6. Fit statistics of the Jiang et al. (2005)'s taper equation for different combinations of random-effects parameters

Random parameters	AIC (smaller is better)	BIC (smaller is better)
None[a]	11017	11047
b_1	11062	11081
b_2	11274	11292
b_3	9887	9905
b_4	10151	10169
b_1, b_2	11024	11049
b_1, b_3	11349	11374
b_1, b_4	9773	9798
b_2, b_3	10275	10300
b_2, b_4	**9642**	**9667**
b_3, b_4	9966	9990

AIC is the Akaike's information criterion, *BIC* is the Bayesian's information criterion [a]Fixed-effects model.

Table 7. Estimated parameters and fit statistics for the selected mixed-effects model

	Estimate	SE[a]
Fixed-effects parameters		
b_1	19.7344	0.1995
b_2	6.9183	0.4035
b_3	0.7554	0.0048
b_4	2.2442	0.0505
Variance components		
σ^2	1.3554	0.0376
Var (b_2)	11.6228	2.6104
Var (b_4)	0.2868	0.0383
Cov (b_2, b_4)	0.3446	0.2244
Goodness-of-fit statistics		
AIC	9642	
BIC	9667	

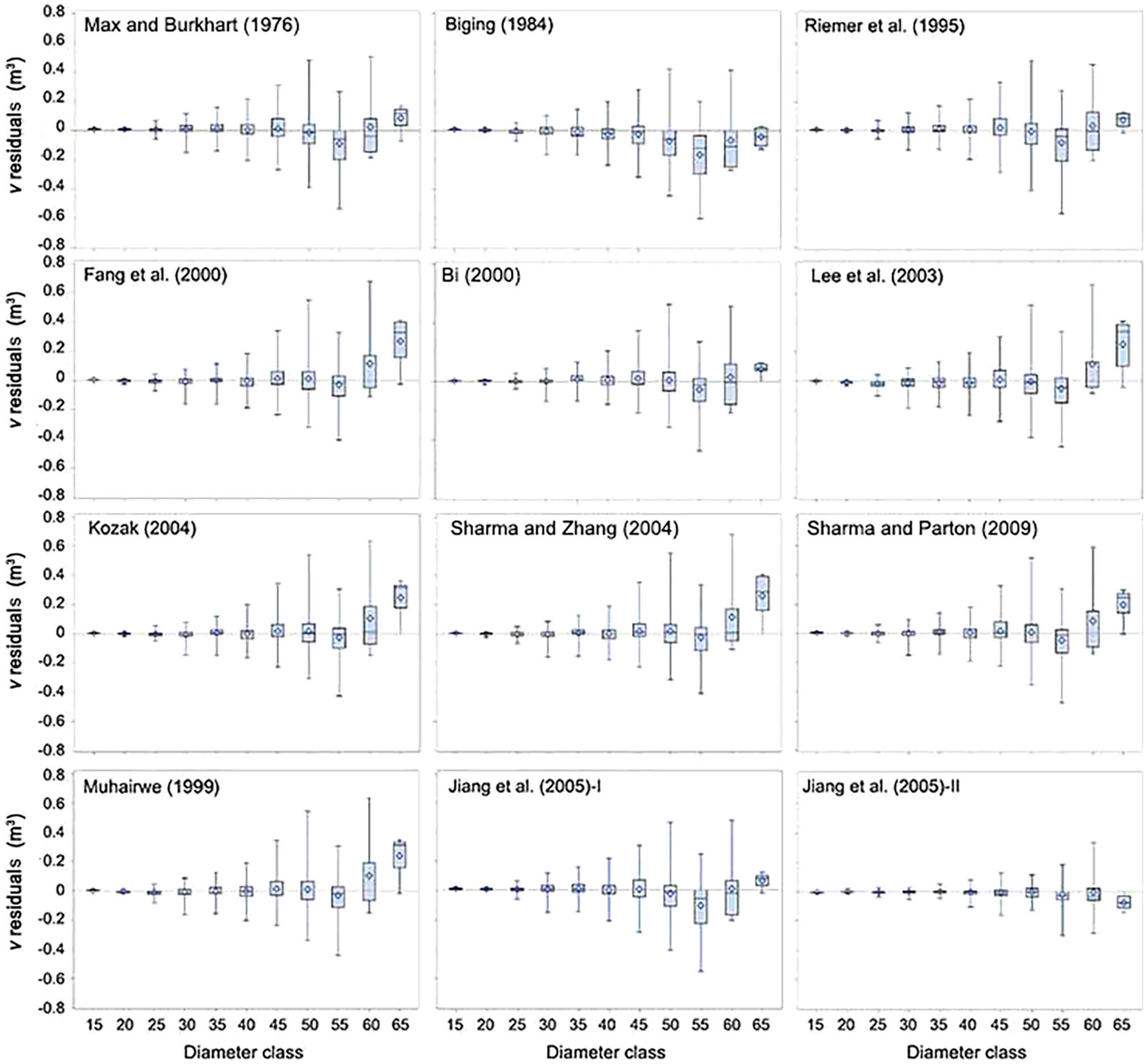

Fig. 7. Box plots of merchantable volume (v) residuals versus diameter classes for the models

taper models in stem diameter and total stem volume estimations. More than 98% and 99% of the total variance in diameter and total stem volume estimations, respectively, can be explained by the taper models. The RMSE values ranged between 1.12–1.62 cm and 0.02–0.07 m³ for diameter and total stem volume estimations, respectively.

As indicated by many researchers, it is well known that the taper model fitted by nonlinear mixed-effects modeling approach can improve the goodness-of-fit statistics compared to the ordinary least squares. Compared with the results of OLS, the mixed-effect modeling approach significantly improved prediction performance of the taper model in this study. However, for prediction purposes, many authors suggest using the fixed-effects models in the absence of calibration data (de-Miguel et al., 2012; Arias-Rodil et al., 2015; Hussain et al., 2020). The fixed-effects models are more accurate in the predictions when random parameters of the mixed models are supposed to be

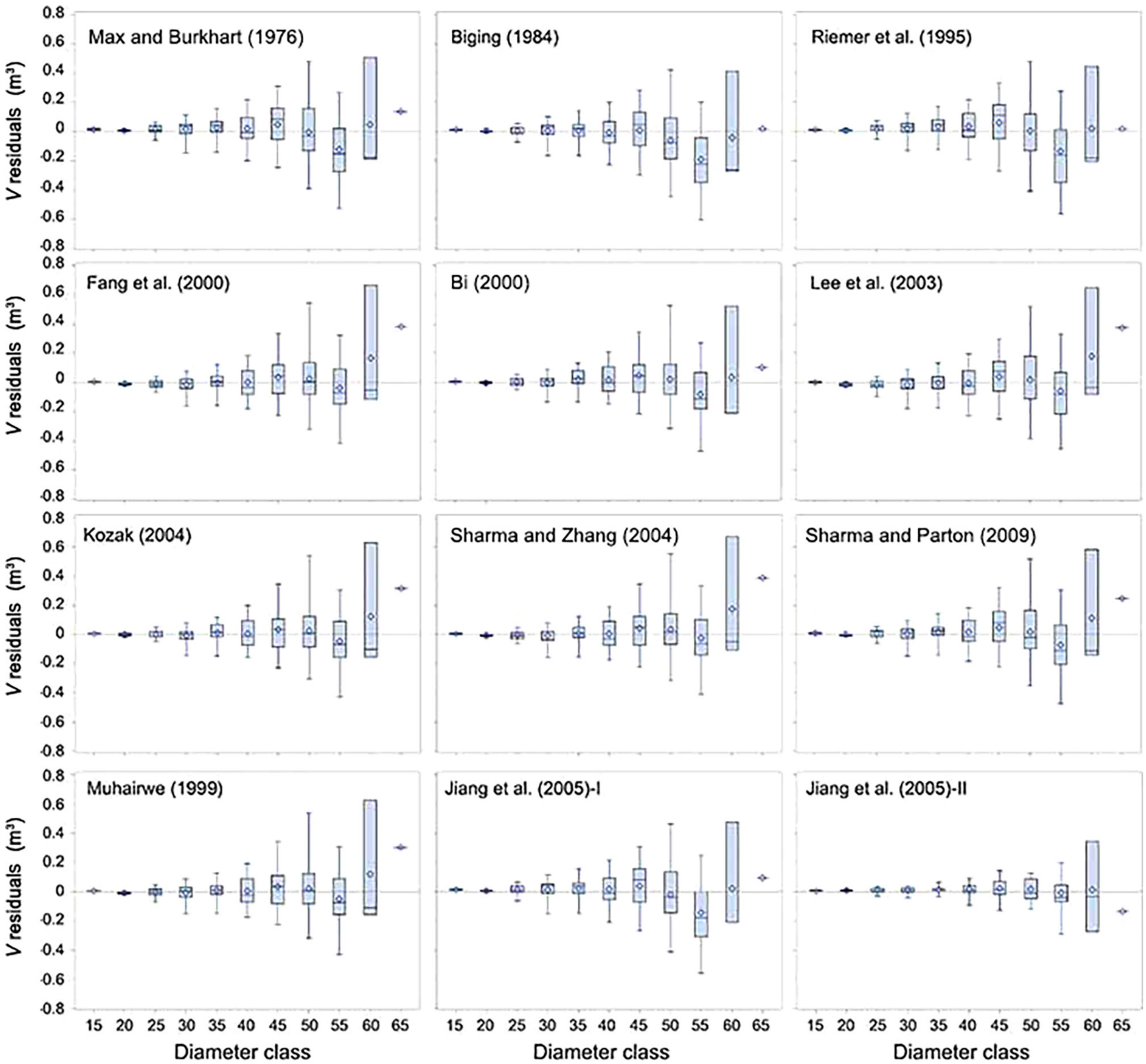

Fig. 8. Box plots of total volume (*V*) residuals versus diameter classes for the models

Table 8. Fit statistics for Jiang et al. (2005)-II with fixed, fixed part of mixed, and mixed model

Models	Diameter				Total volume			
	R^2	RMSE	AIC	MAD	R^2	RMSE	AIC	MAD
Fixed	0.9839	1.6138	2791	1.1310	0.9929	0.0665	−854	0.0399
Fixed part of mixed	0.9837	1.6238	2827	1.1395	0.9927	0.0674	−850	0.0393
Mixed	0.9922	1.1221	681	0.7654	0.9994	0.0193	−1247	0.0134

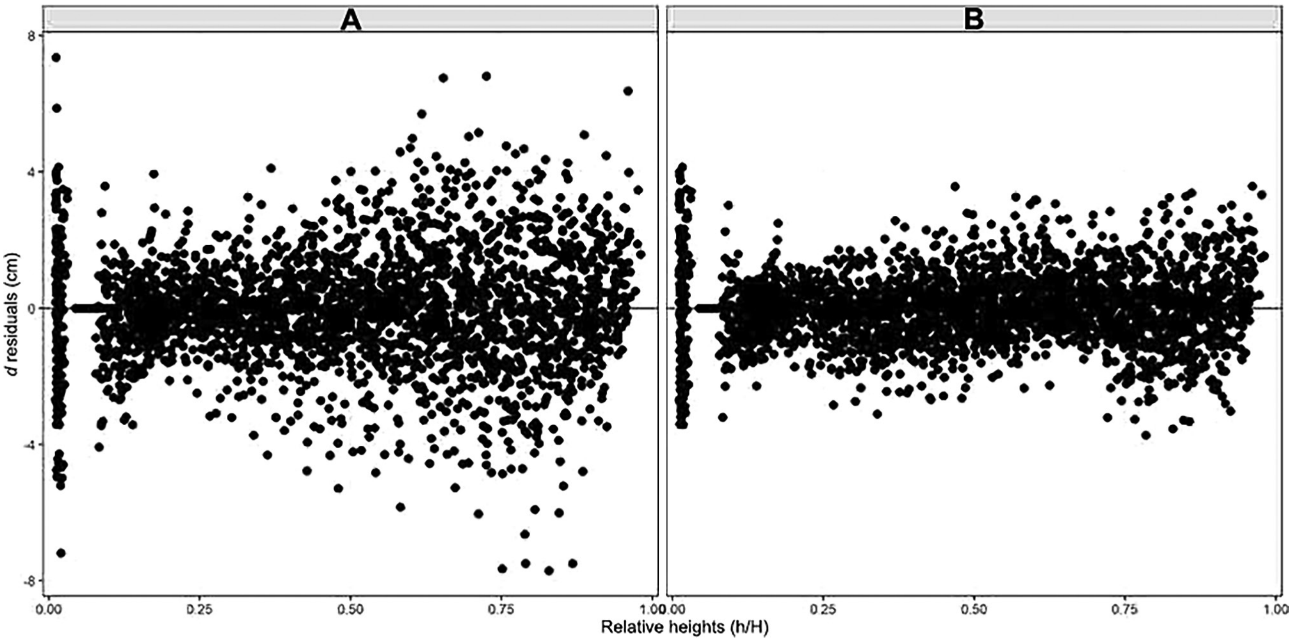

Fig. 9. Residuals for the fixed-effect and mixed-effect model by relative stem height

zero, and additional measurements are not available for model calibration (Guzmán et al., 2012; de-Miguel et al., 2012).

Since calibration may be a feasible option in some cases, de-Miguel et al. (2012) advised reporting both fixed and mixed-effect forms of a model. Accordingly, mixed-effects modeling was carried out for Jiang et al. (2005)-II, which improved the model accuracy.

Discussion

This study analyzed 12 commonly used taper models for reliable estimations of stem diameter, merchantable volume, height, and total stem volume for natural and pure Brutian pine trees. To address the autocorrelation issue, a continuous autoregressive error structure was used to solve the models that arise when multiple consecutive measurements are used for each tree to develop stem taper models. The models analyzed did not show any significant multicollinearity problems, except for a few (Muhairwe, 1999; Bi, 2000; Max & Burkhart, 1976). As noted by Crecente-Campo et al. (2009), the multicollinearity does not cause a problem in the practical use of the models. It was observed that the most successful results for the estimation of stem diameter, merchantable volume, height, and total volume were obtained with the models of Jiang et al. (2005), Bi (2000), and Kozak (2004), respectively, while the least successful estimations were obtained with Biging (1984), Riemer et al. (1995), and Lee et al. (2003), respectively. Similar results were reported by Hussain et al. (2021a) for Korean pine and Manchurian pine in Northeast China and by Hussain et al. (2021b) for white birch and costata birch species in Northeast China. Hussain et al. (2020) for some pine species in Northeast China, Özçelik and Crecente-Campo (2016) for cedar species in Türkiye, and Özçelik & Brooks (2012) for five primary tree species in Türkiye. Although in the study conducted by Li & Weiskittel (2010), the reduced model of Clark et al. (1991) was not the most successful model for diameter estimations, it produced the best results for volume estimations in their study. Although Hussain et al. (2020) reported that the model of Max & Burkhart (1976) was as good as the models of Kozak (2004) and Fang et al. (2000) in terms of estimation performance for three important pine species in Northeast China, the model of Max & Burkhart (1976) showed poor estimation performance in this study. Another model analyzed in this study is the model of Jiang et al. (2005). This model is a reduced form of the model of Clark et al. (1991). Although the Jiang et al. (2005) stem diameter model has been used relatively less in studies analyzing stem diameter models for different tree species, it produced the most successful results for stem diameter and volume estimations in studies conducted by Özçelik & Brooks (2012), Şenyurt et al. (2017), and Saygili & Kahriman (2023) for different regions and tree species in Türkiye. Although the model of Fang et al. (2000) produced successful results for volume estimation and stem diameter in many studies (Diéguez-Aranda et al. 2006; Corral-Rivas et al., 2007; Crecente-Campo et al., 2009; López-Sánchez et al., 2016; Özçelik et al., 2016; Shahzad et al., 2020; Li et al., 2024), it ranked in the middle in terms of estimation performance in this study. Clearly, the model of Jiang et al. (2005) excels at predicting stem merchantable and total volumes. In this study, the Jiang et al. (2005) model reduced the RMSE by 51% and 49% when compared to the next best equation for merchantable and total stem volumes, respectively. Further, it does not require numerical integration like the variable exponent equations. However, it should be noted that the application of Jiang et al. (2005) model requires additional measurement of diameter at 5.30 m above ground. This measurement is not typically obtained in a regular forest inventory. Hussain et al. (2020) found that even when the diameter value at 5.30 m was estimated using the model proposed by Clark et al. 1991, this model showed better estimation performance than the other analyzed models. However, in this study, the use of interpolated diameter at 5.30 m substantially increased the error statistics values produced by this model.

The most successful models for the tree species used in this study when this value could not be obtained were those of Bi (2000) and Kozak (2004), respectively. The difference between the diameter and volume estimation performances of these two models was very small, and they were not found to be superior to each other in terms of them prefer ability when their relative ranks were taken into account. Both models showed similar error distributions for diameter and volume estimations for relative height classes and diameter classes, respectively. However, the model of Bi (2000) showed a more homogeneous and smaller error distribution than the model of Kozak (2004) for large trees in volume estimations. Hussain et al. (2020) obtained the most successful results for three pine species (Dahurian larch, Korean spruce, and Manchurian fir) in northeastern China with the models of Kozak (2004) and Fang et al. (2000) after the model of Clark et al. (1991). However, in the same study, the models of Lee et al. (2003) and Bi (2000) were found to have the lowest predictive performance for diameter and volume estimations, respectively. In the study conducted by Schröder et al. (2015), the model of Bi (2000) showed lower estimation performance than the other models analyzed. de-Miguel et al. (2012) found the model of Bi (2000) to be the second-best model for diameter and volume estimation. In the study conducted by Wilms et al. (2024), the model of Kozak (2004) was found to be the most successful model for diameter estimation with and without bark, followed by Lee et al. (2003) and Muhairwe (1999). In this study, the model of Lee et al. (2003) was found to be one of the most unsuccessful models for diameter and volume estimation, while the model of Muhairwe (1999) ranked in the middle in terms of estimation performance. Consistent with the results of this study, Hussain et al. (2021b) mentioned that the model of Lee et al. (2003) was the least successful one for estimating stem diameter. Ulak et al. (2022) analyzed the success of stem diameter estimations using seventeen different stem diameter models for a tropical tree species in Nepal and found that the most successful estimations were obtained with the model of Sharma & Zhang (2004). However, this model was one of the least successful models analyzed in terms of estimation performance in this study.

Figure 5 presents that all the analyzed models produced higher estimation errors in stem diameter estimations for relative heights between 0–10% and 65–85% compared to other relative height classes. As stated by Jiang et al. (2005), 0–10% of the tree height is the region where the stem swelling is the highest and the diameters are the most unevenly distributed among the trees. Again, 60–65% of the tree height is the region where branching starts, and therefore the variability in stem form is the most intense. For all models analyzed, the success in diameter estimation was higher for all relative height classes except for the 0–10% and 65–85% relative height classes of the tree stem. It can be said that the diameter estimation performance of all models in the lower parts of the stem, except for the first 0–10% of the stem, is more successful than the upper stem parts. This is especially important since the 50% and lower parts of the tree stem are more commercially and economically valuable. Shahzad et al. (2020) reported similar results for white birch species in Northeast China, Crecente-Campo et al. (2009) for native Scots pine species in Spain, and Corral-Rivas et al. (2007) for pine species in the Northwest Durango region of Mexico. Hussain et al. (2020) for pine species in Northeast China, Hussain et al. (2021a) for Korean pine and Manchurian pine in Northeast China, Schröder et al. (2015) for slash pine plantations, Diéguez-Aranda et al. (2006) for Scots pine plantations in Northeast Spain, Özçelik & Crecente-Campo (2016) for native cedar trees in Türkiye, and Özçelik & Brooks (2012) for five primary tree species in Türkiye. All models analyzed tended to estimate smaller diameters for relative heights above 90%. However, since the tip of the stem is relatively low-valued and carries a lower volume value than other parts of the stem, it does not have a large effect on the models' estimation performance (Crecente-Campo et al., 2009; Schröder et al., 2015).

When the distribution of the errors produced by the model of Jiang et al. (2005) is examined in terms of diameter classes in Figure 8, it is seen that they have a very low error variance up to 50 cm, unlike the other analyzed models. Analyzing the results given in Table 5 and Table 6, the model of Jiang et al. (2005) produced more successful results for volume estimation than diameter estimation. All other analyzed models were more successful in diameter estimations than volume estimations. Fortin et al. (2013) stated that this interesting situation may be due to the fact that the diameter value required for the breast surface at any point on the stem is obtained by conversion from the diameter values estimated along the stem. In the study by Li & Weiskittel (2010), the model of Kozak (2004) was the most successful one for diameter estimations, whereas the most successful models for volume estimations were those of modified form of Clark et al. (1991) and Fang et al. (2000). Similarly, in the study conducted by Schröder et al. (2015), Kozak (1988; 2004) models were the most successful models for diameter estimations with and without bark, while Max & Burkhart (1976) was the most successful model for volume estimations. Therefore, when selecting a stem diameter model, it is essential that the evaluation is performed for both diameter and volume estimations (Li & Weiskittel, 2010; Fortin et al., 2013; Schröder et al., 2015).

Evaluating all the results together, the most appropriate models to estimate height, stem diameter, total volume, and merchantable volume is the model of Jiang et al. (2005). It can be recommended for estimating the diameter at any height of the tree stem and merchantable and total stem volume in order to make more accurate estimations for the natural Brutian pine species in the Akhisar Region.

Considering the results of the studies conducted to date, it was observed that when stem diameter models were improved by applying the nonlinear mixed-effects modeling (NLME) approach, the evaluation criteria improved significantly compared to OLS. After deciding on the model of Jiang et al. (2005) being the most successful stem diameter model for the reasons mentioned above, various combinations of random-effects parameters were used to create nonlinear mixed-effects models. Table 6 presents the results of comparing the constructed mixed-effects models using AIC and BIC values. As mentioned earlier, one of the major advantages of the NLME approach is that it allows calibration of the model in the presence of prior information. However, as suggested by many researchers (Meng et al., 2009; de-Miguel et al., 2012; López-Sánchez et al., 2016; Sabatia & Burkhart, 2015; Arias-Rodil et al., 2015; Hussain et al., 2020a; Hussain et al., 2020b), it is recommended to use fixed-effect models in the absence of prior information for model calibration. de-Miguel et al. (2012), Arias-Rodil et al. (2015), and Shahzad et al. (2020) stated that fixed-effect models are more successful in diameter and volume estimation in the absence of prior information for calibration and if the random parameters of the mixed-effect model are assumed to be zero.

The most successful random-effect parameter combination is b_2 and b_4. Of these parameters, defines the lower part of the stem (1.3–5.3 m) and defines the middle and upper part of the stem (Hussain et al., 2020). With a higher value, the parameter b_4 has a major impact on the model, and the parameter b_3 contributes a little. Therefore, the parameter b_4 can explain the tree-level variation in both sections. The variation is usually minimum in the lower stem (Fig. 5). It is seen that the addition of random effects even to only one of these parameters ($b_4 + u_2$) positively affects the estimation performance of the model in the middle and upper parts of the tree stem, and the addition of two random effects (($b_2 + u_1$) and ($b_4 + u_2$)) positively affects the estimation performance in the lower, middle and upper parts of the tree stem. Figure 9 illustrates that with the addition of random effects to the parameters and , the error distribution in diameter estimations for the model's relative height classes is both reduced and more homogeneous for all stem parts compared to the fixed-effect model.

Conclusion

Taper equations were developed to estimate stem diameter, height, merchantable volume, and total stem volume for natural Brutian pine species in Akhisar region of western Türkiye. Although some stem profile models have been developed for Brutian pine stands in southern Türkiye, this is the first time that taper models have been developed for Brutian pine stands in western Türkiye. The taper equations developed in this study will help to make more realistic volume estimations and stem diameter for Brutian pine in the region. 12 stem diameter models were analyzed in the study, and we obtained the most successful results for diameter, height, merchantable volume, and total stem volume estimations with the model of Jiang et al. (2005). The Jiang et al. (2005) model has been used much less frequently than other models in studies analyzing stem diameter models. The results obtained showed that diameter, height, merchantable volume, and total volume estimations can be made successfully with the model of Jiang et al. (2005) for Brutian pine species. However, it is necessary to know the diameter of the trees at a height of 5.30 m to use both models. Therefore, the models of Bi (2000) and Kozak (2004) can be recommended when this is not possible. Furthermore, the addition of random effects to the model within the mixed-effects model approach improved the estimation performance of the model, reduced the inter- and intra-tree error variance and made it more homogeneous. Although the data used in this study covers the range of diameter and heights typically expected in Brutian pine, the data were collected from one region. Thus, the models should be applied outside this region cautiously.

Authorship contribution

Ramazan Özçelik: Conceptualization, Methodology, Investigation, Validation, Software, Formal Analysis, Writing – review & editing

Krishna P. Poudel: Methodology, Investigation, Validation, Software, Formal Analysis, Writing – review & editing

Mehmet Denizhan Balcı: Conceptualization, Investigation, Validation, Data curation Writing – review & editing.

Şerife Kalkanlı Genç: Conceptualization, Investigation, Validation, Software, Formal Analysis, Writing – review & editing.

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Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that might appear to have influenced the work reported in this paper.

References

Akaike H (1974) A new look at statistical model identification. IEEE Transactions on Automatic Control 19: 716–723. doi:10.1109/TAC.1974.1100705.

Alkan O & Özçelik R (2020) Stem taper equations for diameter and volume predictions of *Abies cilicica* Carr. in the Taurus Mountains, Turkey. Journal of Mountain Science 17: 3054–3069. doi:10.1007/s11629-020-6071-x.

Arias-Rodil M, Diéguez-Aranda U, Rodríguez Puerta F, López-Sánchez CA, Canga Líbano, Cámara Obregón A & Castedo-Dorado F (2015) Modelling and localizing a stem taper function for *Pinus radiata* in Spain. Canadian Journal of Forest Research 45: 647–658. doi:10.1139/cjfr-2014-0276.

Belsey DA (1991) Conditioning diagnostics, collinearity and weak data in regression. John Wiley & Sons Inc., New York.

Bi H (2000) Trigonometric variable-form taper equations for Australian eucalyptus. Forest Science 46: 397–409. doi:10.1093/forestscience/46.3.397.

Biging GS (1984) Taper equations for second-growth mixed conifers in northern California. Forest Science 30: 1103–1117. doi:10.1093/forestscience/30.4.1103.

Brooks JR, Jiang L & Özçelik R (2008) Compatible stem volume and taper equations for Brutian pine, Cedar of Lebanon, and Cilicica fir in Turkey. Forest Ecology and Management 256: 147–151. doi:10.1016/j.foreco.2008.04.018.

Burkhart HE & Tomé M (2012) Modeling forest trees and stands. Springer.

Castedo-Dorado F, Gómez-García E, Diéguez-Aranda U, Barrio-Anta M & Crecente-Campo F (2012) Aboveground stand-level biomass estimation: a comparison of two methods for major forest species in northwest Spain. Annals of Forest Science 69: 735–746. doi:10.1007/s13595-012-0191-6.

Crecente-Campo F, Rojo Alboreca A & Diéguez-Aranda U (2009) A merchantable volume system for *Pinus sylvestris* L. in the major mountain ranges of Spain. Annals of Forest Science: 66: 1–12. doi:10.1051/forest/2009078.

Clark III A, Souter RA & Schlaegel BE (1991) Stem profile equations for southern tree species. USDA Forest Service, Southeastern Forest Experiment Station Research Paper SE-282. Asheville, NC.

Corral-Rivas J, Diéguez-Aranda U, Corral Rivas S & Castedo-Dorado F (2007) A merchantable volume system for major pine species in El Salto, Durango (Mexico). Forest Ecology and Management 238: 118–129. doi:10.1016/j.foreco.2006.09.074.

de-Miguel S, Mehtätalo L, Shater Z, Kraid B & Pukkala T (2012) Evaluating marginal and conditional predictions of taper models in the absence of calibration data. Canadian Journal of Forest Research 42: 1383–1394. doi:10.1139/x2012-090.

Diéguez-Aranda U, Castedo-Dorado F, Álvarez-González JG & Rojo A (2006) Compatible taper function for Scots pine plantations in northwestern Spain. Canadian Journal of Forest Research 36: 1190–1205. doi:10.1139/x06-008.

Fang Z, Borders BE & Bailey RL (2000) Compatible volume-taper models for loblolly and slash pine based on a system with segmented stem form factors. Forest Science 46: 1–12. doi:10.1093/forestscience/46.1.1.

Fonweban JB, Gardiner B & Auty D (2012) Variable-top merchantable volume equations for Scots pine (*Pinus sylvestris*) and Sitka spruce (*Picea sitchensis* (Bong.) Carr.) in Northern Britain. Forestry: An International Journal of Forest Research 85: 237–253. doi:10.1093/forestry/cpr069.

Fortin MR, Schneider & Saucier JP (2013) Volume and error variance estimation using integrated stem taper models. Forest Science 59: 345–358. doi:10.5849/forsci.11-146.

Garber SM & Maguire DA (2003) Modeling stem taper of three Central Oregon species using nonlinear mixed effects models and autoregressive error structures. Forest Ecology and Management 179: 507–522. doi:10.1016/S0378-1127(02)00528-5.

GDF (2015) Forest Resources. The General Directorate of Forests, Ankara.

Guzmán G, Pukkala T, Palahí M & de-Miguel S (2012) Predicting the growth and yield of *Pinus radiata* in Bolivia. Annals of Forest Science 69: 335–343. doi:10.1007/s13595-011-0162-3.

He P, Hussain A, Shahzad MK, Jiang L & Li F (2021) Evaluation of four regression techniques for stem taper modeling of Dahurian larch (*Larix gmelinii*) in Northeastern China. Forest Ecology and Management 494: 119336. doi:10.1016/j.foreco.2021.119336.

He P, Jiang L & Li F (2022) Evaluation of parametric and non-parametric stem taper modeling approaches: A case study for *Betula platyphylla* in Northeast China. Forest Ecology and Management 525: 120535. doi:10.1016/j.foreco.2022.120535.

Hussain A, Shahzad MK, He P & Jiang L (2020) Stem taper equations for three major conifer species of Northeast China. Scandinavian Journal of Forest Research 35: 562–576. doi:10.1080/02827581.2020.1843703.

Hussain A, Shahzad MK, Burkhart HE & Jiang L (2021a) Stem taper functions for white birch (*Betula platyphylla*) and costata birch (*Betula costata*) in the Xiaoxing'an Mountains, Northeast China. Forestry: An International Journal of Forest Research 94: 714–733. doi:10.1093/forestry/cpab014.

Hussain A, Shahzad MK, Jiang L & Li F (2021b) Segmented taper models form for Manchurian fir and Korean spruce in northeastern China. Cerne 27: e-102659. doi:10.1590/01047760202127012659.

Jiang L, Brooks JR & Wang J (2005) Compatible taper and volume equations for yellow-poplar in West Virginia. Forest Ecology and Management 213: 399–409. doi:10.1016/j.foreco.2005.04.006.

Kozak A (1988) A variable-exponent taper equation. Canadian Journal Forest Research 18: 1363–1368. doi:10.1139/x88-213.

Kozak A (1997) Effects of multicollinearity and autocorrelation on the variable-exponent taper functions. Canadian Journal Forest Research 27: 619–629. doi:10.1139/x97-011.

Kozak A & Kozak RA (2003) Does cross validation provide additional information in the evaluation of regression models? Canadian Journal Forest Research 33: 976–987. doi:10.1139/x03-022.

Kozak A (2004) My last words on taper equations. The Forest Chronicle 80: 507–515. doi:10.5558/tfc80507-4.

Lee WK, Seo JH, Son YM, Lee KH & von Gadow K (2003) Modeling stem profiles for *Pinus densiflora* in Korea. Forest Ecology and Management 172: 69–77. doi:10.1016/S0378-1127(02)00139-1.

Li R & Weiskittel AR (2010) Comparison of model forma for estimating stem taper and volume in the primary conifer species of the North American Acadian Region. Annals of Forest Science 67: 302. doi:10.1051/forest/2009109.

Li R, Weiskittel AR, Dick AR, Kershaw JA & Seymour RS (2012) Regional stem taper equations for eleven conifer species in the Acadian Region of North America: development and assessment. Northern Journal of Applied Forestry 29: 5–14. doi:10.5849/njaf.10-037.

Li D, Jia W, Guo H, Sun Y & Wang F (2024) Compatible taper and volume systems for *Larix olgensis* and *Larix kaempferi* in northeast China. European Journal of Forest Research 143: 65–79. doi:10.1007/s10342-023-01611-7.

Liang R, Sun Y, Zhou L, Wang Y, Qiu S & Sun Z (2022) Analysis of various crown variables on stem form for *Cunninghamia lanceolata* based on ANN and taper function. Forest Ecology and Management 507: 119973. doi:10.1016/j.foreco.2021.119973.

López-Sánchez CA, Rodríguez-Soalleiro R, Castedo-Dorado F, Corral-Rivas S & Álvarez-González JG (2016) A taper function for *Pseudotsuga menziesii* plantations in Spain. Southern Forests: A Journal of Forest Science 78: 131–135. doi:10.2989/20702620.2015.1136505.

Max TA & Burkhart HE (1976) Segmented polynomial regression applied to taper equations. Forest Science 22: 283–289. doi:10.1093/forestscience/22.3.283.

McTague JP & Weiskittel AR (2020) Evolution, history, and use of stem taper equations: a review of their development, application, and implementation. Canadian Journal of Forest Research 51: 210–235. doi:10.1139/cjfr-2020-0326.

Meng SX, Huang S, Yang Y, Trincado G & VanderSchaaf CL (2009) Evaluation of population-averaged and subject-specific approaches for modeling the dominant or codominant height of lodgepole pine trees. Canadian Journal of Forest Research 39: 1148–1158. doi:10.1139/X09-039.

Muhairwe CK (1999) Taper equations for *Eucalyptus pilurаris* and *Eucalyptus grandis* for the north coast in New South Wales, Australia. Forest Ecology and Management 113: 251–269. doi:10.1016/S0378-1127(98)00431-9.

Návar J, Rodríguez-Flores FJ & Domínguez-Calleros P (2013) Taper functions and merchantable timber for temperate forests of northern Mexico. Annals of Forest Research 56: 165–178. doi:10.15287/afr.2013.51.

Newnham RM (1988) A variable-form taper function. Petawawa National Forestry Institute. PI-X-83.

Özçelik R & Brooks JR (2012) Compatible volume and taper models for economically important tree species of Turkey. Annals of Forest Science 69: 105–118. doi:10.1007/s13595-011-0137-4.

Özçelik R & Crecente-Campo F (2016) Stem taper equations for estimating merchantable volume of Lebanon cedar trees in the Taurus Mountains, Southern Türkiye. Forest Science 62: 78–91. doi:10.5849/forsci.14-212.

Özçelik R & Karaer K (2016) Development of merchantable volume equations for natural brutian pine and black pine stands in Eğirdir District. Journal of the Faculty of Forestry Istanbul University 66: 59–74. doi:10.17099/jffiu.24073.

Özçelik R, Karatepe Y, Gürlevik N, Cañellas I & Crecente-Campo F (2016) Development of ecoregion-based merchantable volume systems for *Pinus brutia* Ten. and *Pinus nigra* Arnold. in southern Turkey. Journal of Forestry Research 27: 101–117. doi:10.1007/s11676-015-0147-4.

Pancoast A (2018) Evaluation of taper and volume estimation techniques for ponderosa pine in eastern Oregon and eastern Washington. Master Thesis, Oregon State University.

Parresol BR & Thomas CE (1989) A density-integral approach to estimating stem biomass. Forest Ecology and Management 26: 285–297. doi:10.1016/0378-1127(89)90089-3.

Poorter H, Niklas KJ, Reich PB, Oleksyn J, Poot P & Mommer L (2012) Biomass allocation to leaves, stems and roots: meta-analyses of interspecific variation and environmental control. The New Phytologist 193: 30–50. doi:10.1111/J.1469-8137.2011.03952.X.

Poudel KP & Cao QV (2013) Evaluation of methods to predict Weibull parameters for characterizing diameter distributions. Forest Science 59: 243–252. doi:10.5849/forsci.12-001.

Poudel KP, Özçelik R & Yavuz H (2020) Differences in stem taper of black alder (*Alnus glutinosa* subsp. *barbata*) by origin. Canadian Journal of Forest Research 50: 581–588. doi:10.1139/cjfr-2019-0314.

Qadir N & Poudel KP (2025) Differences in stem taper of loblolly pine (Pinus taeda) grown in Coastal Plains and Southern Appalachians regions of the United States. Canadian Journal of Forest Research 55: 1–11. doi:10.1139/cjfr-2025-0155.

Riemer T, Gadow Kv & Sloboda B (1995) Ein Modell zur beschreibung von Baumschäften. Allgemeine Forst-und Jagdzeitung 166: 144–147.

Rodríguez F, Lizarralde I, Fernandez-Landa A & Condes S (2014) Non-destructive measurement techniques for taper equation development: a study case in the Spanish Northern Iberian Range. European Journal of Forest Research 133: 213–223. doi:10.1007/s10342-013-0739-5.

Sabatia CO & Burkhart HE (2015) On the use of upper stem diameters to localize a segmented taper equation to new trees. Forest Science 61: 411–423. doi:10.5849/forsci.14-039.

Sakici OE, Misir N, Yavuz H & Misir M (2008) Stem taper functions for *Abies nordmanniana* subsp. *bornmulleriana* in Turkey, Scandinavian Journal of Forest Research 23: 522–533. doi:10.1080/02827580802552453.

Saygili B & Kahriman A (2023) Modeling compatible taper and stem volume of pure Scots pine stands in Northeastern Turkey. İForest - Biogeosciences and Forestry 16: 38–46. doi:10.3832/ifor4099-015.

SAS Institute Inc (2008) SAS/ETS® 9.2 User's guide, SAS Institute Inc, Cary, N.C.

Schröder T, Costa EA, Valério AF & Santos Lisboa G (2015) Taper Equations for Pinus elliottii Engelm. in Southern Paraná, Brazil. Forest Science 61: 311–319. doi:10.5849/forsci.14-054.

Şenyurt M, Ercanlı İ & Bolat F (2017) Taper equations based on nonlinear mixed effect modeling approach for *Pinus nigra* in Çankırı forests. Bosque 38: 545–554. doi:10.4067/S0717-92002017000300012.

Shahzad MK, Hussain A & Jiang L (2020) A model form for stem taper and volume estimates of Asian white birch (*Betula platyphylla*): a major commercial tree species of Northeast China. Canadian Journal of Forest Research 50: 274–286. doi:10.1139/cjfr-2019-0088.

Shahzad MK, Hussain A, Burkhart HE, Li F & Jiang L (2021) Stem taper functions for *Betula platyphylla* in the Daxing'an Mountains, northeast China. Journal of Forestry Research 32: 529–541. doi:10.1007/s11676-020-01152-4.

Sharma M & Zhang SY (2004) Variable-exponent taper equations for jack pine, black spruce, and balsam fir in eastern Canada. Forest Ecology and Management 198: 39–53. doi:10.1016/j.foreco.2004.03.035.

Sharma M & Parton J (2009) Modeling stand density effects on taper for jack pine and black spruce plantations using dimensional analysis. Forest Science 55: 268–282. doi:10.1093/forestscience/55.3.268.

Sharma M (2024) Taper equations for trembling aspen trees grown in natural origin mixed stands. Ontario Ministry of Natural Resources, Science and Research Branch, Peterborough, ON. Science and Research Technical Report, TR-64.

Sun Y, Jia W & Saidahemaiti S (2024) An additive model system for heartwood, sapwood and bark diameter – A working example in *Pinus koraiensis* Siebold & Zucc. Plantations. Computers and Electronics in Agriculture 220: 108868. doi:10.1016/j.compag.2024.108868.

Tassisa G & Burkhart HE (1998) An application of mixed effects analysis to modeling thinning effects on stem profile of loblolly pine. Forest Ecology and Management 103: 87–101. doi:10.1016/S0378-1127(97)00179-5.

Ulak S, Ghimire K, Gautam R, Bhandari SK, Poudel KP, Timilsina YP, Pradhan D & Subedi T (2022) Predicting the upper stem diameters and volume of a tropical dominant tree species. Journal of Forestry Research 33: 1725–1737. doi:10.1007/s11676-022-01458-5.

Wilms F, Berendt F, Bronisz K, Bashutska U, Fotelli M, Radoglou K & Spyroglou G (2024) Applying taper function models for black locust plantations in Greek post-mining areas. Scientific Reports 14: 13557. doi:10.1038/s41598-024-63048-1.

Yang S-I & Radtke PJ (2022) Predicting bark thickness with one- and two-stage regression models for three hardwood species in the southeastern US. Forest Ecology and Management 503: 119778. doi:10.1016/j.foreco.2021.119778.

Yang S-I & Qiao Y (2024) Quantifying bark thickness and bark volume with alternative modeling procedures for eight species in the southeastern US. Forest Ecology and Management 553: 121631. doi:10.1016/j.foreco.2023.121631.